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Applied Probability

Frank A. Haight

*The Pennsylvania State University
University Park, Pennsylvania*

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For my dear children, Julian and Molly

Preface

Probability (including stochastic processes) is now being applied to virtually every academic discipline, especially to the sciences. An area of substantial application is that known as operations research or industrial engineering, which incorporates subjects such as queueing theory, optimization, and network flow.

This book provides a compact introduction to that field for students with minimal preparation, knowing mainly calculus and having “mathematical maturity.” Beginning with the basics of probability, the development is self-contained but not abstract, that is, without measure theory and its probabilistic counterpart.

Although the text is reasonably short, a course based on this book will normally occupy two semesters or three quarters. There are many points in the discussions and problems which require the assistance of an instructor for completeness and clarity.

The book is designed to give equal emphasis to those applications which motivate the subject and to appropriate mathematical techniques. Thus, the student who has successfully completed the course is ready to turn in either of two directions: towards direct study of research papers in operations research, or towards a course in abstract probability, for which this text provides the intuitive background.

Pennsylvania State University

Frank A. Haight

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