Numerical
Computation
of Electric
and Magnetic
Fields

Second Edition

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Numerical Computation of Electric and Magnetic Fields

Second Edition

Charles W. Steele

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PREFACE

Since the first edition of this book was published in 1987, there have been several important changes in the state of numerical field computation, as discussed in the Introduction. These changes have motivated the publication of this second edition.

As with the first edition, the objective of this second edition is to give the newcomer to field computation the information needed to perform practical field computations. Again, clarity of presentation is given greater emphasis than a high degree of sophistication or the state of the art. And again, the basic concepts of field computation are presented as well as the commonly used algorithms.

Several persons have provided much valuable information for this second edition. I wish to thank Professor Giorgio Molinari of the University of Genoa, Italy for advice regarding adaptive mesh generation; Dr. C. R. E. Emson of Vector Fields, Ltd., England and Dr. John Brauer of McNeal-Schwendler Corp. for their advice on transient eddy current computation; and Dr. Zoltan Cendes of Ansoft Corp. for information about their adaptive mesh generator.

Again, I would like to acknowledge the support for this second edition by my wife, Candace. Again, I could not have written this book without her support.

PREFACE TO THE FIRST EDITION

For well over a decade, the numerical approach to field computation has been gaining progressively greater importance. Analytical methods of field computation are, at best, unable to accommodate the very wide variety of configurations in which fields must be computed. On the other hand, numerical methods can accommodate many practical configurations that analytical methods cannot. With the advent of high-speed digital computers, numerical field computations have finally become practical.

However, in order to implement numerical methods of field computation, we need algorithms, numerical methods, and mathematical tools that are largely quite different from those that have been traditionally used with analytical methods. Many of these algorithms have, in fact, been presented in the large number of papers that have been published on this subject in the last two decades. And to some of those who are already experienced in the art of numerical field computations, these papers, in addition to their own original work, are enough to give them the knowledge that they need to perform practical numerical field computations.

But newcomers to the art of numerical field computations need a more orderly presentation of the necessary background information than is available to them in these papers. The objective of this book is precisely to provide this orderly presentation of the necessary information. Clarity of presentation is given greater emphasis than a high degree of sophistication or being up to the state of the art. It is my hope that those who read this book will then be able to read and understand the recent papers and thereby achieve the level of sophistication to which they aspire.

Specifically, this book has the objective of presenting information in two different categories:

1. Certain algorithms that have been used successfully for computation of fields.
2. Background information that will help readers to develop their own algorithms.

Putting it another way, the intent of this book is to help readers develop or select the algorithms to be used in these field computations. These algorithms

can then be used in computer programs, and these computer programs can be run in high-speed digital computers to compute the fields.

To build these computer programs, the programmer requires knowledge in a number of other areas beyond the algorithms that are discussed in this book. These areas include computer programming, methods for numerical solution of systems of equations, methods for automatic deployment of node points, and construction of finite elements and computer graphics (to display the computed fields). These important topics are discussed in technical journals as well as in other books.

Any numerical field computation is built upon logic that is necessarily an intimate mixture of mathematics, numerical analysis, and electromagnetic theory. This book is just such a mixture. It is assumed that the reader has some prior knowledge of electromagnetic theory, vector analysis, and linear algebra.

This book covers static and quasi-static field problems in media that generally vary in permittivity, permeability, and conductivity. It does not cover dynamic problems, such as antennas, scatterers, radiation in free space, and waveguide modes. These items are topics for books in themselves.

Here we consider only problems in which both the observer of the fields and the medium throughout the entire domain are fixed with respect to a single frame of reference. That is, we do not consider problems in which a portion of the medium moves with respect to the remainder of the medium (and the field is computed in both)—for example, problems in which the field is to be computed in both the moving rotor and stator of an electric motor or generator. Such problems are an important study in themselves.

An objective of this book has been to use standard nomenclature as much as possible. All of the equations related to electric and magnetic fields are based upon the use of the SI system of units. The symbols used are, for the most part, taken from the international recommendations for quantities, units, and their symbols, as published by the United States of America Standards Institute.

I am deeply indebted to a number of people for their contributions to this book. I appreciate my discussion with Mr. Charles Trowbridge, of Rutherford Laboratories, United Kingdom, regarding my approach to Chapter 5. Dr. Alvin Wexler of the University of Manitoba made valuable suggestions related to my presentation of certain of his material in Chapter 7. In addition, Mr. Trowbridge and Dr. Wexler provided thorough technical reviews of my completed manuscript. Dr. Dennis Lindholm of Ampex Corporation reviewed the majority of the manuscript and made many valuable suggestions. Mrs. Bernie Jones of Palo Alto, California, did an excellent job of typing a large portion of the manuscript.

Finally, and most important, I wish to acknowledge my wife, Candace. She has gently tolerated my work on this book over four annual vacations and

countless weekends. Without her unfailing support and encouragement I could not have written it.

If the reader wishes to communicate with the author regarding this book, he can send the communication to the publisher, who will then forward it to the author.

CHARLES W. STEELE