
Physics of Strained Quantum Well Lasers

PHYSICS OF STRAINED QUANTUM WELL LASERS

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Preface

When this publisher offered me the opportunity to write a book, some six years ago, I did not hesitate to say yes. I had just spent the last four years of graduate school struggling to understand the physics of strained quantum well lasers, and it seemed to me the whole experience was much more difficult than it should have been. For although many of the *results* I needed were easy to locate, the underlying physical premises and intervening steps were not. If only I had a book providing the *derivations*, I could have absorbed them and gone on my way.

Such a book lies before you. It provides a unified and self-contained description of the essential physics of strained quantum well lasers, starting from first principles whenever feasible. The presentation I have chosen requires only the standard introductory background in quantum mechanics, solid state physics, and electromagnetics expected of entering graduate students in physics or electrical engineering. A single undergraduate course in each of these subjects should be more than sufficient to follow the text. More advanced material on quantum mechanics is developed and collected in the first chapter. When possible, I have presented the results in a general setting and have later applied them to specific cases of interest. I find this the most satisfying way to approach the subject, and it has the additional benefit of solving many problems once and for all.

The book focuses on a few topics that, based on the experience of myself and others, are the most difficult to learn from the existing literature. In particular, I discuss the bandstructure of bulk and quantum well semiconductors at length, deriving the tight binding and $\mathbf{k} \cdot \mathbf{p}$ models from scratch. I also devote much space to optical transition rates, paying special attention the sums over continuum states. I have taken great pain to present the intermediate steps in these calculations, and to emphasize what the symbols actually *mean*. Proficiency in these topics should grant the reader immediate access to current literature, both in the form of journal articles and in more advanced, but often less detailed, books. I have tried to reference several of these, especially when the contributions have been comparatively recent. But I have not attempted to provide a catalog of current research or a history of the field. Rather, my

goal was to provide in-depth and accessible coverage of the most challenging and fundamental concepts.

Any book, I now realize, represents an enormous undertaking. I could not have accomplished this alone. Professors J. Singh and K. T. Hecht instructed me in many of these topics, and conversations with Y. Lam, V. Sankaran, B. Jogai, F. Pollack, Y. C. Chang, J. Pamulapati, K. Humphreys, M. Cremer, S. Fiete, D. B. Lemersal Jr., and N. Tufnel were also instrumental. Special thanks are owed to C. F. Beckner III, who provided a well-timed and much-needed infusion of capital in the earliest stages of the project. And I thank my wife Beth, one of the most loving, capable, and good-humored people I have ever met, for her support. Her talents ranged over scheduling, proofreading, moral encouragement, deadline enforcement, and the making of Rice Krispie treats, all of which became critical during the hectic final months when the manuscript was finalized. I would also like to thank John Bodt at Kluwer Academic Publishers. Without his kind support and encouragement, and his quite reasonable insistence that I actually *finish* the book, this manuscript would never have appeared. And S. A. Feld quickly, competently, and graciously provided illustrations on demand.

But there is one person whose contributions to this project dwarf all others. R. E. Sherriff, physicist, computer programmer, guitarist and grammarian, unix-hater, \LaTeX novice, and frequent victim of the King's Gambit took it upon himself to edit, check, proofread, and typeset every page of the manuscript. All without being asked, and at ungodly hours in the morning. He showed such intense concern for the final quality of the document, and argued so forcefully for his editorial prejudices—many of which required substantial revisions or outright rewrites—that I often found myself yielding to his demands out of sheer fright. I am embarrassed to admit what this book would have become without his efforts. For the service of friendship he provided me throughout the duration of the project, and especially during the harrowing final month, I shall remain forever in his debt.

J. P. LOEHR

To Rocky Sherriff, I could not
have asked for a better
colleague and friend