

**LINEAR PROGRAMMING**  
**A Modern Integrated Analysis**

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**INTERNATIONAL SERIES IN  
OPERATIONS RESEARCH & MANAGEMENT SCIENCE**

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# **LINEAR PROGRAMMING A Modern Integrated Analysis**

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**Springer Science+Business Media, LLC**

**Library of Congress Cataloging-in-Publication Data**

Saigal, Romesh.

Linear programming : a modern integrated analysis / Romesh Saigal.

p. cm. -- (International series in operations research & management science ; 1)

Includes bibliographical references and index.

ISBN 978-1-4613-5977-7 ISBN 978-1-4615-2311-6 (eBook)

DOI 10.1007/978-1-4615-2311-6

1. Linear programming. I. Title. II. Series.

T57.74.S23 1995

519.7'2--dc20

95-34605  
CIP

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Originally published by Kluwer Academic Publishers in 1995

Softcover reprint of the hardcover 1st edition 1995

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**Dedicated to**

Shailesh, Ashima, David and Veena

# Contents

<b>Preface</b>	<b>ix</b>
<b>1 Introduction</b>	<b>1</b>
1.1 The Problem . . . . .	1
1.2 Prototype Problems . . . . .	2
1.3 About this Book . . . . .	4
1.4 Notes . . . . .	5
<b>2 Background</b>	<b>7</b>
2.1 Real Analysis . . . . .	7
2.2 Linear Algebra and Matrix Analysis . . . . .	13
2.3 Numerical Linear Algebra . . . . .	20
2.4 Convexity and Separation Theorems . . . . .	27
2.5 Linear Equations and Inequalities . . . . .	31
2.6 Convex Polyhedral Sets . . . . .	36
2.7 Nonlinear System of Equations . . . . .	44
2.8 Notes . . . . .	64
<b>3 Duality Theory and Optimality Conditions</b>	<b>67</b>
3.1 The Dual Problem . . . . .	67
3.2 Duality Theorems . . . . .	70
3.3 Optimality and Complementary Slackness . . . . .	73
3.4 Complementary Pair of Variables . . . . .	78
3.5 Degeneracy and Uniqueness . . . . .	80
3.6 Notes . . . . .	83
<b>4 Boundary Methods</b>	<b>85</b>
4.1 Introduction . . . . .	85
4.2 Primal Simplex Method . . . . .	87
4.3 Bounded Variable Simplex Method . . . . .	98

4.4	Dual Simplex Method . . . . .	103
4.5	Primal - Dual Method . . . . .	107
4.6	Notes . . . . .	110
<b>5</b>	<b>Interior Point Methods</b>	<b>111</b>
5.1	Primal Affine Scaling Method . . . . .	112
5.2	Degeneracy Resolution by Step-Size Control . . . . .	139
5.3	Accelerated Affine Scaling Method . . . . .	148
5.4	Primal Power Affine Scaling Method . . . . .	166
5.5	Obtaining an Initial Interior Point . . . . .	198
5.6	Bounded Variable Affine Scaling Method . . . . .	200
5.7	Affine Scaling and Unrestricted Variables . . . . .	203
5.8	Dual Affine Scaling Method . . . . .	206
5.9	Primal-Dual Affine Scaling Method . . . . .	211
5.10	Path Following or Homotopy Methods . . . . .	222
5.11	Projective Transformation Method . . . . .	244
5.12	Method and Unrestricted Variables . . . . .	252
5.13	Notes . . . . .	262
<b>6</b>	<b>Implementation</b>	<b>265</b>
6.1	Implementation of Boundary Methods . . . . .	265
6.2	Implementation of Interior Point Methods . . . . .	270
6.3	Notes . . . . .	304
<b>A</b>	<b>Tables</b>	<b>307</b>
	<b>Bibliography</b>	<b>315</b>
	<b>Index</b>	<b>338</b>

# Preface

During the past decade, linear programming has gone through a dramatic change. The catalyst for this change is the path breaking work of Narinder K. Karmarkar who, in 1984, presented a polynomial time algorithm generating a sequence of points in the interior of the polyhedron. This method contrasts with the earlier state-of-the art methods that generate a sequence of points on the boundary of the polyhedron. The most notable, and the earliest of these is the simplex method of George B. Dantzig, produced in 1947. The new methods that proceed through the interior of the polyhedron avoid the combinatorial structure of the boundary, which the boundary methods must necessarily contend with. A consequence of this is that even though many of the interior point methods are shown to be polynomial, an example produced in 1971 by Victor L. Klee and George J. Minty demonstrates that in the worst case, simplex method can take an effort which grows exponentially in the data of the problem. Thus, for large problems, the interior point methods may be advantageous. A growing body of computational experience appears to support this conclusion.

Interior point methods trace their roots to the work of I. I. Dikin in 1967. His method is called the (primal) affine scaling method, and was rediscovered by researchers at A. T. & T. Bell Telephone Laboratories while implementing the ideas of Karmarkar in a system called KORBX. Several variants of this method, like the dual and the primal-dual were also generated in the process. Since then, many new methods like the sliding objective function methods, the path-following barrier methods, and potential reduction methods have been generated. Several successful implements of these also exist in the public domain.

This book presents both the boundary and the interior point methods in a unified manner. Approach taken here involves the use of duality theory, in particular the complementary slackness conditions, to derive both the boundary and interior point methods. Most text books on linear programming derive the duality theorems after presenting the computational



aspects of the simplex method. And the theorems of the alternative (like Farkas' lemma) are then derived as corollaries to the duality theorems. This approach is unsatisfactory for our purpose. The approach taken here is to prove Farkas' lemma and other theorems of the alternative as convex separation theorems, and then derive the duality theorems directly from this lemma. Thus the tedium of the simplex method is avoided. In addition several concepts like the existence of strict complementary solutions are generally derogated to an exercise at the end of the duality chapter. These results play an important role in interior point methods, and are thus proved explicitly in the duality chapter of this book.

The author has been teaching and developing the methodology for presenting these ideas in several courses taught at the University of Michigan in Ann Arbor, Michigan and the Naval Postgraduate School in Monterey, California and has paid special attention to the mathematical background required for the study of interior point methods. To study the boundary methods, it is sufficient to have a good background in linear algebra, numerical linear algebra and polyhedral convexity. The study of recent methods, in addition, requires sufficient background in real analysis, convexity and separation theorems as well as an understanding of the methodology for solving nonlinear systems of equations, specially Newton's method and homotopy or global Newtons' methods and their implementations. Thus a chapter presenting this background material has been included in this manuscript.

This book is organized into six chapters. Chapter 1 introduces the problem and some standard formulations to orient the reader to linear programming. In the second chapter, careful attention is paid to the necessary mathematical background required for the study of these methods. The third chapter presents the duality theorem and also the important results on complementary slackness conditions. The fourth chapter presents the three important variants of the simplex method, the primal, the dual and the primal dual methods. The fifth chapter presents the interior point methods. It covers the affine scaling methods, the projective transformation methods, the barrier methods as well as the primal-dual homotopy or global Newton methods. Finally the chapter six is devoted to the implementation of both the interior and the boundary methods. Sparsity preserving and stable methods for the boundary and interior methods are also discussed in this chapter.

This book can be used as a text in a one or two semester advanced graduate course in linear programming. A good two semester sequence could cover sections 2.4 - 2.6, terminating with the proof of Farkas' Lemma. Then after proving the duality theorems of Chapter 3, establish the pri-

mal simplex method. Then cover Chapter 5 and establish the primal and dual affine scaling method, and the path following method. During the first semester, cover only the convergence of boundary and interior methods with the non-degeneracy assumption. During the second semester, prove the convergence of these methods under degeneracy, establish the dual simplex method, primal-dual boundary methods; and the power affine scaling method. During this semester, also cover the implementations of Chapter 6.

The writing of this book would never have been completed without encouragement, help and comments from several people. This is gratefully acknowledged from John R. Birge, Steve Chen, S.-C. Fang, Hans Kremers, Masakazu Kojima, Prashant M. Kulkarni, Masakazu Muramatsu, Shinji Mizuno, Sarat Puthenpura, Krishnamachari S. Ramprasad, Lakshman P. Sinha, Manmohan S. Sodhi, Samer Takriti, Takashi Tsuchiya and Terrance C. Wagner. I am also grateful to the editor Gary Folven of Kluwer Academic Publishers, for his kind words and constant encouragement. Some of the research presented in this book was supported by the grant number CCR 9321550 from the National Science Foundation. I thank NSF (and the project director S. Kamal Abdali) for this support.

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June 1, 1995

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