
**GEOMETRIC METHOD FOR
STABILITY OF NON-LINEAR
ELASTIC THIN SHELLS**

GEOMETRIC METHOD FOR STABILITY OF NON-LINEAR ELASTIC THIN SHELLS

by

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To our Parents and Children

TABLE OF CONTENTS

PREFACE..... *xi*
ACKNOWLEDGEMENT.....*xiii*

CHAPTER 1. Postcritical Deformations of Thin Anisotropic Shells

1.1. Geometric Method in the Nonlinear Theory of
Thin Shells..... 1
1.1.1. Postcritical Deformations of Convex Shells..... 10
1.1.2. Stability Loss of Strictly Convex Shells..... 12
1.1.3. Stability Loss of Convex Developable Shells..... 13
1.2. Asymptotic Form of the Postcritical Deformation
Energy of Elastic Anisotropic Shells..... 16
1.3. Postcritical Deformations of Shallow Strongly
Convex Orthotropic Shells..... 25
1.3.1. Lower Critical Load for Spherical Shells (Caps)
under External Pressure..... 40
1.4. Cylindrical Orthotropic Shells under Axial
Compression..... 42
1.5. Mechanical Interpretation of the Berger’s
Hypothesis for the Global Stability of Statically
Loaded Anisotropic Shells..... 58

**CHAPTER 2. Postcritical Deformations of Thin Elastic
Anisotropic Shells with Linear Memory**

2.1. Introduction..... 65
2.2. Variational Principle A for Thin Elastic
Anisotropic Shells with Linear Memory..... 66
2.3. Postcritical Deformations of Thin Elastic
Orthotropic Cylindrical Shells with Linear
Memory under Uniform External Pressure..... 71
2.3.1. Linear Effect of the Kernel Parameter γ 74

2.4.	Postcritical Deformations of Thin Orthotropic Cylindrical Shells with Linear Memory. Nonlinear Effect of a Kernel Parameter γ	78
CHAPTER 3. Variational Principle for Global Stability of Elasto-Plastic Thin Shells		
3.1.	Introduction.....	87
3.2.	Asymptotic Expression for the Energy of Postcritical Deformations of Elasto-Plastic Shells.....	88
3.2.1.	Variational Principle A for Elasto-Plastic thin shells.....	95
3.3.	Postcritical Behavior of Thin Cylindrical Elasto-Plastic Shells under Axial Compression.....	101
CHAPTER 4. Instability of Thin Elastic and Elasto-Plastic Orthotropic Shells under Combined Static and Dynamic Loading		
4.1.	Introduction.....	109
4.2.	Asymptotic Analysis of Nonlinear Partial Differential Dynamic Equations for Thin Elastic Anisotropic Shells.....	116
4.3.	Cylindrical Orthotropic Shells under Combined Axial Compression Loading.....	124
4.4.	Cylindrical Orthotropic Shells under Combined Uniform External Pressure Loading	132
4.5.	Cylindrical Orthotropic Shells under Static Axial Compression and Short-Duration Dynamic Impulse of External Pressure.....	139
4.6.	Strictly Convex Orthotropic Shells under Combined Dynamic Loading. Expression for the Postcritical Deformation Energy.....	144
4.7.	Dynamic Instability of Strictly Convex Elastic Orthotropic Shells under Combined External Pressure Loading. Critical Parameters of the Process.....	148

4.8. Appendix to Section 4.4..... 155

4.9. Dynamic Instability of Cylindrical Elasto-
Plastic Shells Subjected to Combined Axial
Compression Loading..... 159

CHAPTER 5. Crushing of Plastic Cylindrical Shells Sensitive to the Strain Rate under Axial Impact

5.1. Introduction.....169

5.2. Mathematical Modelling of the
Crushing Process.....171

5.3. Axisymmetric (Concertina) Crushing
Mode 172

5.3.1. Asymmetric (Diamond) Crushing
Mode176

5.3.2. Mixed (Transitive) Crushing
Mode177

5.4. Theoretical Method

5.4.1. Axisymmetric (Concertina) Crushing
Mode178

5.4.2. Asymmetric (Diamond) Crushing
Mode183

5.4.3. Mixed Crushing Mode..... 193

5.5. Characteristics Independent of the
Crushing Mode.....193

5.5.1. Characteristics of the Asymmetric (Diamond)
Crushing Mode.....193

5.5.2. Characteristics of the Mixed Crushing
Mode.....195

5.6. Comparison between Theoretical and
Experimental Data.....196

CHAPTER 6. Appendices

6.1. Introduction.....205

- 6.2. Special Isometric Transformations of Cylindrical Surfaces.....206
 - 6.2.1. Isometric transformation of Cylindrical Surfaces with Periodic Structure.....207
 - 6.2.2. Isometric transformation of Cylindrical Surfaces with Helical Symmetry.....209
 - 6.2.3. Isometric transformation of Cylindrical Surfaces Satisfying Boundary Conditions at the Shell edge.....211
 - 6.2.4. Extension of the Isometric Transformation of Cylindrical Surfaces with Periodic Structure to the Case of Axial Mass Impact.....213
 - 6.2.5. Isometric Transformation of Convex Surfaces.....215
- 6.3. Some Information from the Theory of Surfaces.....217

REFERENCES.....227

INDEX.....239

PREFACE

P R E F A C E

This book deals with the new developments and applications of the geometric method to the nonlinear stability problem for thin non-elastic shells. There are no other published books on this subject except the basic ones of A. V. Pogorelov (1966,1967,1986), where variational principles defined over isometric surfaces, are postulated, and applied mainly to static and dynamic problems of elastic isotropic thin shells.

A. V. Pogorelov (Harkov, Ukraine) was the first to provide in his monographs the geometric construction of the deformed shell surface in a post-critical stage and deriving explicitly the asymptotic formulas for the upper and lower critical loads. In most cases, these formulas were presented in a closed analytical form, and confirmed by experimental data.

The geometric method by Pogorelov is one of the most important analytical methods developed during the last century. Its power consists in its ability to provide a clear geometric picture of the postcritical form of a deformed shell surface, successfully applied to a direct variational approach to the nonlinear shell stability problems. Until now most Pogorelov's monographs were written in Russian, which limited the diffusion of his ideas among the international scientific community. The present book is intended to assist and encourage the researches in this field to apply the geometric method and the related results to everyday engineering practice.

Further developments of the geometric method are carried out in this book, and are directed to stability of thin shells in the case of elastic anisotropy, elastic anisotropy with linear memory and elasto-plastic properties of the shell material. Pogorelov's variational principle for elastic isotropic shells is extended here to non-elastic shell and the proper asymptotical analysis of the corresponding nonlinear partial differential equations is performed. This analysis confirms the validity of hypotheses of the geometric method when applied to the considered shells. The postcritical behavior of cylindrical and strongly convex thin shells under static, combined (static and short-time impulse) and axial impact load is fully investigated. The corresponding deformed shell surfaces are approximated by Pogorelov's isometric surfaces. A new isometric surface is constructed in the case of impacted cylindrical shell. The corresponding critical loads are obtained in closed asymptotic analytical forms and compared with experimental data.

These formulas could be used for a rapid estimation of the global stability of nonlinear elastic thin shells.

Readers should have a basic understanding of introductory surface theory, stability of shells and partial differential equations. The book also provides the basis of Pogorelov's construction of the isometric surfaces, approximating the deformed shell surface in postcritical stage.

The book is intended to serve both as a textbook for post-graduate students in structural engineering and applied mathematics, and as a reference monograph for academic and industrial researchers.

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