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Simplicial Global Optimization

Springer

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Preface

Simplicial global optimization focuses on deterministic covering methods for global optimization partitioning the feasible region by simplices. Although rectangular partitioning is used most often in global optimization, simplicial covering has advantages shown in this book. The purpose of the book is to present global optimization methods based on simplicial partitioning in one volume. The book describes features of simplicial partitioning and demonstrates its advantages in global optimization.

A simplex is a polyhedron in a multidimensional space, which has the minimal number of vertices. Therefore simplicial partitions are preferable in global optimization when the values of the objective function at all vertices of partitions are used to evaluate subregions.

The feasible region defined by linear constraints may be covered by simplices and therefore simplicial optimization algorithms may cope with linear constraints in a delicate way by initial covering. This makes simplicial partitions very attractive for optimization problems with linear constraints.

There are optimization problems where the objective functions have symmetries which may be taken into account for reducing the search space significantly by setting linear inequality constraints. The resulted search region may be covered by simplices.

Applications benefiting from simplicial partitioning are examined in the book: nonlinear least squares regression, center-based clustering of data having one feature, and pile placement in grillage-type foundations. In the examples shown, the search region reduced taking into account symmetries of the objective functions is a simplex thus simplicial global optimization algorithms may use it as a starting partition.

The book provides exhaustive experimental investigation and shows the impact of various bounds, types of subdivision, and strategies of candidate selection on the performance of global optimization algorithms. Researchers and engineers will benefit from simplicial partitioning algorithms presented in the book: Lipschitz branch-and-bound, Lipschitz optimization without the Lipschitz constant. We hope the readers will be inspired to develop simplicial versions of other algorithms for global optimization and even use other non-rectangular partitions for special applications.

The book deals with theoretical, computational, and application aspects of simplicial global optimization. It is intended for scientists and researchers in optimization and may also serve as a useful research supplement for Ph.D. students in mathematics, computer science, and operations research.

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Contents

Acronyms

n	Number of variables
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{D}$	Feasible region
ε	Tolerance
x, y, z	Variables
$\mathbf{x}, \mathbf{y}, \mathbf{z}$	Vectors of variables
$f(\mathbf{x})$	Objective function
$\nabla f(\mathbf{x})$	Gradient of objective function $f(\mathbf{x})$
$F(\mathbf{x})$	Lower bounding function
f^*	Global optimum function value
$f(\mathbf{x}_{\text{opt}})$	ε-global optimum
$\mathbf{x}^*$	Global optimum vector
$\mathbf{x}_{\text{opt}}$	ε-global optimum vector
$\mathbb{S}$	Solution (subregion, optimum point)
$\mathbb{T}$	Finite set of points where the objective function value has been evaluated
$\mathbb{I}$	Subregion of feasible region
$\mathbb{L}$	Candidate set
$\lvert\mathbb{L}\rvert$	Cardinality of a candidate set
$\mathbf{v}$	Vertex of subregion
$\mathbb{V}(\mathbb{I})$	Set of vertices of subregion
$\mathbb{O}$	n-dimensional ball
LB	Lower bound for minimum
UB	Upper bound for minimum
R	Circumradius
D	Determinant
p	Number of processors
p, q	Norm index
s_p	Speedup
e_p	Efficiency

$\|\mathbf{x}\|_q$	q-norm, $(q \geq 1)$
$\|\mathbf{x}-\mathbf{y}\|_q$	Distance function
L_p	Lipschitz constant of objective function according to the p-norm
K	Lipschitz constant of derivatives
μ	Simple μ type Lipschitz bound
φ	Piyavskii type bound
ψ	Lipschitz bound based on the radius R of the circumscribed multidimensional sphere
$\mu_2^{1,2,\infty}$	μ_2 type Lipschitz bound with the $1,2$, and ∞ norms
$\varphi^1\psi^2\mu_2^{2,\infty}$	Aggregate bound composed of φ, ψ, and μ_2 type bounds with different norms
$\widehat{\varphi^1\psi^2\mu_2^{2,\infty}}$	Aggregate bound with vertex verification
r_{ψ^2/μ_2^2}	Ratio showing goodness of ψ^2 bound against μ_2^2 bound
$r(f^*)$	Search progress ratio
fe	Number of function evaluations
$t(s)$	Optimization time
TNS	Total number of simplices
MCL	Maximal size of candidate list