

# Lecture Notes in Physics

Volume 877

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Gary P. Zank

# Transport Processes in Space Physics and Astrophysics



Springer

Gary P. Zank  
CSPAR  
University of Alabama in Huntsville  
Huntsville, AL, USA

ISSN 0075-8450 ISSN 1616-6361 (electronic)  
ISBN 978-1-4614-8479-0 ISBN 978-1-4614-8480-6 (eBook)  
DOI 10.1007/978-1-4614-8480-6  
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2013946652

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*For my parents, Pat and Lyn Zank, and  
my daughters, Taryn and Janine*



# Acknowledgment

I should like to thank Adele Corona for her constant encouragement and help with the writing of this book. Without her, it would never have been completed. The help provided by Dr Alex Dosch was invaluable. He made numerous excellent suggestions for the organization, exposition, and content of the book, and I benefited enormously from his help and insight. Alex developed an excellent Solutions Manual that is available from the publisher. I should like to thank my many collaborators and teachers over the years, from whom I learned everything I know. Finally, the contribution of the students who grappled with this class over the past few years has to be acknowledged. They caught typos, made suggestions for clearer presentation of ideas and results, and worked the many problems in the book.





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