Theory of Fuzzy Computation

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Theory of Fuzzy Computation

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To my son Demetrios-Georgios
and my father Georgios

Preface

Nowadays, computing is ubiquitous and pervasive. Many consumer products, like electronic and electric appliances and vehicles, are using some sort of computational device to ensure proper operation. At the same time, fuzzy set theory (roughly, the theory that claims that elements belong to sets to some degree) gains momentum as a basis for the solution of many problems in engineering, technology, medicine, etc. Thus, it seems that some sort of *fuzzy computation* would be a very important development. Indeed, since the inception of fuzzy set theory in the mid-1960s, there have been attempts to define fuzzy conceptual computing devices and so to develop a formal theory of fuzzy computation. These efforts yielded important results quite recently, and they include the discovery that there is no universal fuzzy Turing machine and that fuzzy Turing machines are capable to solve problems that no ordinary Turing machine can solve. Simultaneously, work has been carried out to design and implement fuzzy hardware (e.g., see [70] for a not so recent overview of the field of fuzzy hardware).

Unfortunately, even today many researchers and scholars confuse the notion of fuzzy computing with fuzzy expert systems, fuzzy database systems, fuzzy information management, fuzzy knowledge management, fuzzy e-commerce services, fuzzy web services, etc.—all these are "applications" of fuzzy set theory that are implemented atop normal hardware using ordinary computing tools. In a sense, fuzziness appears to be some sort of linguistic layer that facilitates the expression of certain problems, something completely unacceptable in my own opinion. In different words, there is a gap between what most people think fuzzy computing is about and what it should be about in fact. Now, fuzziness is not a linguistic phenomenon; it is a mathematical model of vagueness, which, in turn, is a characteristic property of this cosmos (Fig. 1 shows what I consider as vagueness in nature). For instance, one may argue that the probabilities employed in quantum mechanics are in fact *possibilities* in the sense of Zadeh's theory of possibilities [147] (e.g., see [40] for an early and brief discussion of this idea). Thus, there are a number of misconceptions that have to be clarified.

This book, among others, is an effort to remedy this deficiency. It presents most, if not all, milestones in the development of what one might call a theory of fuzzy computation. In particular, the first chapter starts by giving a historical overview of computation and fuzziness and gives a preliminary response to the question what is fuzzy computation,

Chapter 2 is a brief overview of the classical theory of computation. It includes a thorough presentation of Turing machines and some of their variations (e.g., multitape and nondeterministic Turing machines). Then, there is an introduction to Kolmogorov–Uspensky algorithms and a discussion of their computational power. This discussion is followed by a

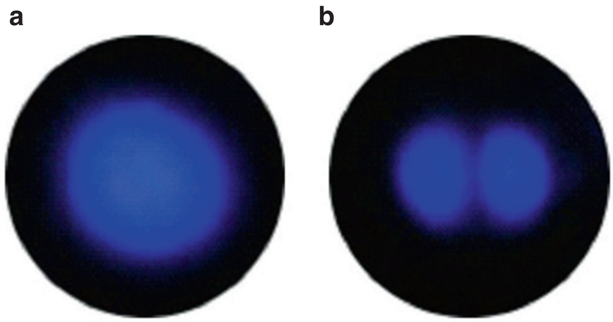

Figure 1: Vagueness in nature—Images of the end atoms of carbon chains. Reprinted figure with permission from I.M. Mikhailovskij, E.V. Sadanov, T.I. Mazilova, V.A. Ksenofontov, and O.A. Velicodnaja, PHYSICAL REVIEW B, 80, 165404, 2009. Copyright (2009) by the American Physical Society

presentation of recursive functions, the analytical hierarchy, and a discussion of the Church–Turing thesis.

The third chapter is an introduction to the theory of fuzzy sets. The chapter starts with some necessary definitions and results from order theory and proceeds with the definition of fuzzy sets and L-fuzzy sets. Next, there is a brief presentation of fuzzy relations, t-norm, and t-conorms. The chapter concludes with some thoughts concerning fuzzy set theory in general.

A thorough presentation of fuzzy Turing machines is given in chapter four. There is also a discussion of fuzzy formal languages, fuzzy recursion theory, fuzzy universality, and the computational power of fuzzy Turing machines.

The fifth chapter presents other models of computation that are inspired by fuzzy set theory. In particular, there is a discussion of fuzzy P systems, fuzzy labeled transition systems, fuzzy X machines, and the fuzzy chemical abstract machine. In addition, there is a discussion of some fuzzy process algebras.

Finally, the book has two appendices—one that discusses Zadeh's idea of *computing with words* and one that introduces rough computing devices. Strictly, computing with words is not a model of computation per se but an idea that is supposed to help people to solve problems where, for example, quantities are not expressed by numbers but by words like "much" or "too little" instead. Rough sets are an alternative approach to describe vagueness that makes use of an upper and a lower approximation. Since this book is about (conceptual) computing devices that are vague and operate in a vague environment, I felt it was more than necessary to include a short appendix on the emerging field of rough computing devices.

Intended Readership The book is self-contained, but the text assumes familiarity with certain mathematical notions (e.g., readers are expected to be familiar with relations and their properties). However, everyone with some elementary mathematical maturity willing to get a thorough understanding of the theory of fuzzy computation as it currently stands can read the book by skipping the difficult parts. Also, the book can also be used as a teaching vehicle for a graduate-level course in fuzzy computability theory. In fact, the author has followed the book's exposition for a short course on the theory of fuzzy computation that was taught at COPPE/UFRJ, the Alberto Luiz Coimbra Institute for Graduate Studies and Research in Engineering of the Federal University of Rio de Janeiro, in June 2012.

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