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Spectral and Dynamical Stability of Nonlinear Waves

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Todd Kapitula: *I am grateful to my colleagues, both here at the college and elsewhere, for their support and encouragement during this endeavor. I am especially thankful to my wife, Laura, for her unwavering support during the seemingly endless rounds of revisions. Finally, I wish to thank Keith for his boundless enthusiasm for, and perseverance with, this project.*

Keith Promislow: *I thank Ronay, Sylvia, Isaac, and Levi for tolerating my absence during the writing of this book, and Todd for tolerating my presence. I am grateful to Roger Temam for teaching me the value of writing books.*

Foreword

Waves, patterns, and other permanent structures have always played a pivotal role in applied mathematics. It is the proverbial no-brainer that they should, and also that their stability to perturbations is an issue of importance. This is the first full-scale book devoted to the stability theory, developed over the past few decades, which lies at the interface of dynamical systems and functional analysis. While functional analysis provides the framework and affords the proper posing of questions, it is dynamical systems that give many of the answers.

The Evans function provides the key bridge between these two theoretical viewpoints, and much of this text concerns its development. John Evans wrote his seminal papers in the 1970s. He had trained as a medical doctor, but became enamored with mathematics just as he was about to embark on his medical career, at least at the point where his financial outlook would have improved dramatically. He gave it all up to pursue a PhD in mathematics, but his mathematical interests still reflected his medical background. Hodgkin–Huxley had formulated the equations governing the propagation of nerve impulses about 20 years earlier, and had computed, using a primitive calculator, travelling waves that represented the propagation of a nerve impulse. It was the discovery of this wave that showed they had found the mechanisms underlying the functioning of a nerve. So, it was known that the wave was present in the equations, but surely it also had to be stable. Showing its stability was the task that Evans set himself. He did not achieve it, but he did build a theory that has had applications far beyond neuroscience.

What happened next was indicative of the fortuitous circumstances that are often involved in research advances. I had heard Evans speak in 1980 while I was a postdoctoral fellow in British Columbia, and realized during his talk that his ideas provided the missing piece to a problem that had interested many of us, namely the stability of the fast traveling pulse of the FitzHugh–Nagumo equations. Although FitzHugh and Nagumo had separately formulated this system as a simplification of the Hodgkin–Huxley equations, Evans himself had resisted looking at this problem as he was after a more general result. In hindsight, had I not been lucky enough to come across this connection, and thus been able to show the power of Evans's ideas, they may never have achieved the prominence and range of applications they now enjoy. It took me most of the 1980s to convince the community of the importance of what I had by then named the Evans function. Gardner was working with me early on, and a major shift occurred when we convinced Alexander to join the effort; in particular, his topological and geometric expertise allowed us to put it in a completely new light. I should also mention Maginu in Japan who had realized early on the power of the Evans function, and his group made a number of contributions.

Simultaneously, the subject of stability was growing up for problems with Hamiltonian structure. To applied mathematicians, the division of physical problems into conservative and dissipative is fundamental, and it is not surprising that a stability theory evolved on both sides. Benjamin's stability result for water waves was perhaps the first result in this area, but much of the stability work on Hamiltonian systems was developed in Russia. Indeed, it was stimulated by the fundamental work of Arnol'd and Zakharov's work on soliton stability. This area also matured in the 1980s: Weinstein resolved the stability of standing waves of the nonlinear Schrödinger equation in his PhD thesis, and Grillakis, Strauss, and Shatah built a comprehensive theory for the stability of waves of a large class of Hamiltonian partial differential equations.

The stability story was greatly enriched by the entwining of these two threads. Of particular note was the work of Pego and Weinstein who showed that the Evans function plays a key role in determining the stability of the Korteweg–de Vries soliton. They showed that many of the calculations carried out in conservative systems could be recast in terms of the Evans function. This insight then led to the resolution of unsolved problems in the stability of wave motion for conservative partial differential equations. Zumbrun led a parallel development in his work on the stability of viscous shocks. This is an area marked by hard analytic estimates, but Zumbrun managed to push it much further through the use of insights originating in the Evans function approach.

But that is all prehistory. This book is about how all this early research grew into a full-blooded mathematical theory and how the importance of these stability methods can now be measured in a large number of applied areas. What began in nerve impulse equations has now had an impact in nonlinear optics, gas dynamics, fluid mechanics, material sciences, combustion theory, and Bose–Einstein condensates, among other areas.

For a graduate student needing to learn this material, a postdoc switching into an area that needs it, or even somebody (like me!) who claims to be expert in this area, this book will be invaluable. It covers an enormous amount of ground and does so with great insight and style. I have often been told that I should have written this book and that I should have done it long ago! I am glad I did not, as Kapitula and Promislow have done a much better job than I ever could have.

Chapel Hill, NC Christopher K.R.T. Jones

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