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Oswaldo Luiz do Valle Costa
François Dufour

Continuous Average Control of Piecewise Deterministic Markov Processes

Springer

Oswaldo Luiz do Valle Costa
Department of Telecommunications
and Control Engineering
University of São Paulo
São Paulo, São Paulo
Brazil

François Dufour
Universite Bordeaux 1, Institut de
Mathématiques de Bordeaux
Institut Polytechnique de Bordeaux
Talence Cedex
France

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Preface

The intent of this book is to present recent results in the control theory for the long-run average continuous control problem of Piecewise Deterministic Markov Processes (PDMPs). This is neither a textbook nor a complete account of the state-of-the-art on the subject. Instead, we attempt to provide a systematic framework for an understanding of the main concepts and tools associated with this problem, based on previous works of the authors. The limited size of a book makes it unfeasible to cover all the aspects in this field and therefore some degree of specialization was inevitable. Due to that, the book focuses mainly on the long-run average cost criteria and tries to extend to the PDMPs some well-known techniques related to discrete-time and continuous-time Markov decision processes, including the so-called "average inequality approach," "vanish discount technique," and "policy iteration algorithm."

Most of the material presented in this book was scattered throughout a variety of sources, which included journal articles and conference proceedings papers. This motivated the authors to write this text, putting together systematically these results. Although the book is mainly intended to be a theoretically oriented text, it also contains some motivational examples. The notation is mostly standard although, in some cases, it is tailored to meet specific needs. A glossary of symbols and conventions can be found at the end of the book.

The book is targeted primarily for advanced students and practitioners of control theory. In particular, we hope that the book will be a valuable source for experts in the field of Markov decision processes. Moreover, we believe that the book should be suitable for certain advanced courses or seminars. As background, one needs an acquaintance with the theory of Markov decision processes and some knowledge of stochastic processes and modern analysis.

The authors are indebted to many people and institutions which have contributed in many ways to the writing of this book. We gratefully acknowledge the support of the IMB, Institut Mathématiques de Bordeaux, INRIA Bordeaux Sud Ouest, team CQFD, and the Laboratory of Automation and Control—LAC/USP at the University of São Paulo. This book owes much to our research partners, to whom we are immensely grateful. Many thanks go also to our former Ph.D. students. We acknowledge with great pleasure the efficiency and support of Donna

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Last, but not least, we are very grateful to our families for their continuing and unwavering support. To them we dedicate this book.

São Paulo, Brazil O. L. V. Costa
Bordeaux, France F. Dufour

Contents

Notation and Conventions

As a general rule, lowercase greek and roman letters are used for functions while uppercase greek are used for selectors. Sets and spaces are denoted by capital roman letters. Blackboard and calligraphic letters represent Borel measurable spaces or a σ-algebra. Sometimes it is not possible or convenient to adhere completely to this rule, but the exceptions should be clearly perceived based on their specific context.

The following lists present the main symbols and general notation used throughout the book, followed by a brief explanation and the number of the page of their definition or first appearance.

Symbol	**Description**
$\square$	End of proof
$\mathbb{N}$	The set of natural numbers
$\mathbb{N}_*$	The set of positive real numbers
$\mathbb{R}$	The real numbers
$\mathbb{R}_+$	The positive real numbers
$\mathbb{R}^d$	The d-dimensional euclidian space
$\overline{\mathbb{R}}_+$	$= \mathbb{R}_+ \cup \{+\infty\}$
$\mathcal{B}(X)$	σ-algebra generated by the open sets of X
$\mathbb{B}(X;Y)$	Borel bounded functions from X into Y
$\mathbb{B}(X)$	$= \mathbb{B}(X;\mathbb{R})$
$\mathbb{B}(X)^+$	$= \mathbb{B}(X;\mathbb{R}^+)$
$\mathcal{M}(X)$	The set of all finite measures on $(X, \mathcal{B}(X))$
$\mathbb{M}(X;Y)$	Borel measurable functions from X into Y
$\mathbb{M}(X)$	$= \mathbb{M}(X;\mathbb{R})$
$\mathbb{M}(X)^+$	$= \mathbb{M}(X;\mathbb{R}^+)$
$\mathbb{B}_g(X)$	Functions υ such that $\sup_{x\in X} \frac{\lvert v(x)\rvert}{g(x)} < +\infty$
$\mathbb{C}(X)$	Continuous functions from X to $\mathbb{R}$
$\mathcal{P}(X)$	Set of all probability measures on $(X, \mathcal{B}(X))$
h^+	The positive part of a function h

h^-	The negative part of a function h
η	The Lebesgue measure on the real numbers
I_A	The indicator function of the set A
E	The state space of a PDMP: an open subset of $\mathbb{R}^n$
∂E	The boundary of the state space E
$\overline{E}$	The closure of the state space E
$\phi(x, t)$	The flow of a PDMP
$t_*(x)$	The time the flow ϕ takes to reach the boundary ∂E starting from x
$\mathcal{X}$	Vector field associated with the flow ϕ
$\mathbb{U}$	The set of control actions
$\mathbb{U}(x)$	The set of feasible control actions that can be taken when the state process is in $x \in \overline{E}$
λ	The jump rate
Q	The transition measure Q
$\mathbb{M}^{ac}(E)$	Functions absolutely continuous along the flow with limit toward the boundary
$\overline{\lambda}$	A upper bound of λ with respect to the control variable
Δ	An arbitrary fixed point in ∂E
K	The set of feasible state/action pairs
$\mathcal{U}$	The class of admissible control strategies
$\widehat{\phi}$	The flow of the controlled PDMP
$\widehat{\lambda}^U$	The jump rate of the controlled PDMP
$\widehat{Q}^U$	The transition measure of the controlled PDMP
$P^U_{\hat{x}}$	The probability of the probability space on which the PDMP is defined
$E^U_{\hat{x}}$	The expectation under the probability $P^U_{\hat{x}}$
$\widehat{X}^U(t)$	The controlled PDMP
$X(t)$	The state of the system
$Z(t)$	The value of $X(t)$ at the last jump time before t
$\tau(t)$	The time elapsed between the last jump and time t
$N(t)$	The number of jumps of the process $\{X(t)\}$ at time t
$(T_n)_{n\in\mathbb{N}}$	The sequence of jump times of the PDMP
$f,\ r$	The running and boundary costs
$\mathbf{J}^\alpha(U,t)$	The finite horizon cost function
$\mathbf{J}(U,t)$	$= \mathbf{J}^0(U, t)$
$p^*(t)$	The counting process associated with the number of times the process hits the boundary up to time t
$\mathcal{A}(U, x)$	The long-run average cost function
$\mathcal{J}_{\mathcal{A}}(x)$	The value function associated with the long-run average cost function
$\mathcal{D}^\alpha(U, x)$	The α-discounted cost function

$\mathcal{J}_{\mathcal{D}}^{\alpha}(x)$	The value function associated with the α-discounted cost function
$\mathcal{D}_m^{\alpha}(U,x)$	The truncated version of the α-discounted cost function
$L^1(\mathbb{R}_+;\mathbb{C}(\mathbb{U}))$	The set of Bochner integrable functions with values in $\mathbb{C}(\mathbb{U})$
$L^{\infty}(\mathbb{R}_+;\mathcal{M}(\mathbb{U}))$	The space of bounded measurable functions from $\mathbb{R}_+$ to $\mathcal{M}(\mathbb{U})$
$\mathcal{V}^r, \mathcal{V}^r(x), \mathcal{V}(x)$	See Definition 2.12
$\mathbb{V}^r$, $\mathbb{V}^r(x)$	The set of relaxed controls
$\mathbb{V}, \mathbb{V}(x)$	The set of ordinary controls
$[\Theta]_t$	Shifted control strategy (see Definition 2.7)
$\mathcal{K}$	The set of feasible state/relaxed-control pairs
$w(x,\mu)$	See equation (2.8)
$Qh(x,\mu)$	See equation (2.9)
$\lambda Qh(x,\mu)$	See equation (2.10)
$\Lambda^{\mu}(x,t)$	See equation (2.11)
$G_{\alpha}(x,\Theta;A)$	See equation (2.12)
$G_{\alpha}h(x,\Theta)$	See equation (2.13)
$L_{\alpha}v(x,\Theta)$	See equation (2.14)
$H_{\alpha}w(x,\Theta)$	See equation (2.15)
$\mathcal{L}_{\alpha}(x,\Theta)$	See equation (2.16)
$G, L, H, \mathcal{L}$	$G=G_0, L=L_0, H=H_0, \mathcal{L}=\mathcal{L}_0$
$\mathcal{T}_{\alpha}(\rho,h)(x)$	The one-stage optimization operator
$\mathcal{R}_{\alpha}(\rho,h)(x)$	The relaxed one-stage optimization operator
$\mathcal{T}$ and $\mathcal{R}$	$\mathcal{T}=\mathcal{T}_0$ and $\mathcal{R}=\mathcal{R}_0$
$\mathcal{S}_{\mathbb{U}}, \mathcal{S}_{\mathbb{V}}, \mathcal{S}_{\mathbb{V}^r}$	The sets of measurable selectors
u_{ϕ}	See Definition 2.22
U_{ϕ}	See Definition 2.23
$J_m^U(t,x,k)$	The truncated version of finite horizon α-discounted cost function
$(f_j)_{j\in\mathbb{N}}$	The approximating sequence of the running cost f
$(r_j)_{j\in\mathbb{N}}$	The approximating sequence of the running cost r
$\underline{\lambda}$	A lower bound of λ with respect to the control variable
$\overline{f}$	A upper bound of f with respect to the control variable
$\widehat{u}(w,h)\in\mathcal{S}_{\mathbb{U}}$	See definition 3.12
$\widehat{u}_{\phi}(w,h)\in\mathcal{S}_{\mathbb{V}}$	See definition 3.12
$\widehat{U}_{\phi}(w,h)\in\mathcal{U}$	See definition 3.12
$\mathcal{W}g$	$=\mathcal{R}_{\alpha}(0,g)$
$h_{\alpha}(x)$	$=\mathcal{J}_{\mathcal{D}}^{\alpha}(x)-\mathcal{J}_{\mathcal{D}}^{\alpha}(x_0)$, the relative difference of the α-discount value functions $\mathcal{J}_{\mathcal{D}}^{\alpha}$
K_h	A lower bound for h_{α}
$\overline{h}$	A upper bound for h_{α}
g	Test function for the so-called expected growth condition
$\overline{r}$	Test function for the so-called expected growth condition
ν_u	Invariant probability measure of the kernel $G(.,u_{\phi};.)$
κ	See equation (4.18)

Abbreviation	**Description**
MDP(s)	Markov Decision Process(es)
PDMP(s)	Piecewise Deterministic Markov Process(es)
PIA	Policy Iteration Algorithm