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Quantum Science and Technology

Aims and Scope

The book series Quantum Science and Technology is dedicated to one of today's most active and rapidly expanding fields of research and development. In particular, the series will be a showcase for the growing number of experimental implementations and practical applications of quantum systems. These will include, but are not restricted to: quantum information processing, quantum computing, and quantum simulation; quantum communication and quantum cryptography; entanglement and other quantum resources; quantum interfaces and hybrid quantum systems; quantum memories and quantum repeaters; measurement-based quantum control and quantum feedback; quantum nanomechanics, quantum optomechanics and quantum transducers; quantum sensing and quantum metrology; as well as quantum effects in biology. Last but not least, the series will include books on the theoretical and mathematical questions relevant to designing and understanding these systems and devices, as well as foundational issues concerning the quantum phenomena themselves. Written and edited by leading experts, the treatments will be designed for graduate students and other researchers already working in, or intending to enter the field of quantum science and technology.

Renato Portugal

Quantum Walks and Search Algorithms

 Springer

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Preface

This is a textbook about *quantum walks* and *quantum search algorithms*. The reader will take advantage of the pedagogical aspects of this book and learn the topics faster and make less effort than reading the original research papers, often written in jargon. The exercises and references allow the readers to deepen their knowledge on specific issues. Guidelines to use or to develop computer programs for simulating the evolution of quantum walks are also available.

There is a gentle introduction to quantum walks in Chap. 2, which analyzes both the discrete- and continuous-time models on a discrete line state space. Chapter 4 is devoted to Grover's algorithm, describing its geometrical interpretation, often presented in textbooks. It describes the evolution by means of the *spectral decomposition* of the evolution operator. The technique called *amplitude amplification* is also presented. Chapters 5 and 6 deal with analytical solutions of quantum walks on important graphs: line, cycles, two-dimensional lattices, and hypercubes using the Fourier transform. Chapter 7 presents an introduction of quantum walks on generic graphs and describes methods to calculate the limiting distribution and the mixing time. Chapter 8 describes spatial search algorithms, in special a technique called *abstract search algorithm*. The two-dimensional lattice is used as example. This chapter also shows how Grover's algorithm can be described using a quantum walk on the complete graph. Chapter 9 introduces Szegedy's quantum-walk model and the definition of the quantum hitting time. The complete graph is used as example. An introduction to quantum mechanics in Chap. 2 and an appendix on linear algebra are efforts to make the book self-contained.

Almost nothing can be extracted from this book if the reader does not have a full understanding of the postulates of quantum mechanics, described in Chap. 2, and the material on linear algebra described in the appendix. Some extra bases are required: It is desirable that the reader has (1) notions of quantum computing, including the circuit model, references are provided at the end of Chap. 2, and (2) notions of classical algorithms and computational complexity. Any undergraduate or graduate student with this background can read this book. The first five chapters are more amenable to reading than the remaining chapters and provide a good basis for the area of quantum walks and Grover's algorithm. For those who have strict interest

in the area of quantum walks, Chap. 4 can be skipped and the focus should be on Chaps. 2, 5–7. Grover’s algorithm plays an essential role in Chaps. 8 and 9. Chapter 6 is very technical and repetitive. In a first reading, it is possible to skip the analysis of quantum walks on finite lattices and hypercubes in Chap. 6 and in the subsequent chapters. In many passages, the reader must go slow, perform the calculations and fill out the details before proceeding. Some of those topics are currently active research areas with strong impact on the development of new quantum algorithms.

Corrections, suggestions, and comments are welcome, which can be sent through webpage (qubit.lncc.br) or directly to the author by email (portugal@lncc.br).

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This book would not have started without an inner motivation, on which my family has a strong influence. I thank Cristina, João Vitor, and Pedro Vinicius. They have a special place in my heart.

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