Nonlinear Systems and Complexity

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Newtonian Nonlinear Dynamics for Complex Linear and Optimization Problems

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To Luciana and José Luis with whom
I enjoyed many fierce pinball matches
in El Escorial and Forte dei Marmi,

Luis Vázquez

To my beloved ones,

Salvador Jiménez

Preface

In a *Pinball Machine*, the player tries to score points by manipulating a metal ball on a playing field inside a glass covered case. The objectives of the game are to score as many points as possible, to earn free games and to maximize the time spent playing by earning extra balls and keeping balls in play as long as possible. Apart from the new challenging features, the good old pinball playing field is essentially a planar surface inclined upwards from 3 to 7 °, away from the player, and include multiple targets and scoring objectives. The ball is put into play by the use of the *plunger* which propels upwards the ball. Once the ball is in play, it tends to move downwards towards the player, although the ball can move in any direction, sometimes unpredictably, as the result of contact with objects on the playing field or by the player actions. To return the ball to the upper part of the playing field, the player makes use mainly of one or more *flippers*. The game ends whenever the ball crosses downwards the "flippers barrier" [26].

The *Pinball Machine* provides a simple mechanical example of the linear optimization problem, basically in a surface embedded in the three-dimensional space. In all pinball games, the play with every ball finishes when that ball reaches the minimum gravitational potential energy immediately after the flippers barrier. On the other hand, the playing field where the ball moves is, effectively, a convex planar region. The duration of the ball motion is always finite, even considering the human interaction. This fact indicates that the minimum of the *Objective Function* (in this case, the Potential Gravitational Energy) is always attained by the motion of the ball. This example suggests us to associate the solutions of some optimization problems to the motion of Newtonian particles. At the same time, this example is a bridge that allows us to construct algorithms for linear/nonlinear optimization problems and unconstrained extrema by applying to them the numerical algorithms used to simulate the equations of motion for a Newtonian particle. These are the motivation and the objective of this monograph. The framework of this monograph wants to be constructive: we want to present some methods and their features that show how Newton's equation for the motion of one particle in classical mechanics combined with finite difference methods can create a mechanical scenario within which we may solve some basic, though complex, problems. We, thus, apply these ideas to solve linear systems and eigenvector problems, as well as programming, both linear and nonlinear, in different dimensions, in the spirit of the suggestive books by Mordecai Avriel [3] and John T. Betts [5]. For this latter case, the goal of the monograph is to show a breakthrough analysis method of optimization by combining the features of the motion of a Newtonian classical particle and finite difference numerical algorithms associated with the equation of motion. Many challenging questions remain open, but we think that a new, fresh and feasible approach to solve them is shown.

This unified numerical and mechanical approach is new, to the best of our knowledge, and we believe that our view represents a simple but useful tool not yet fully exploited.

This monograph is intended for a broad public: undergraduate and graduate students or researchers who are confronted in their work with linear systems and eigenvalue or optimization problems and who are open to new perspectives in the way these problems can be addressed. To help the reader to explore these ideas, we propose a list of related exercices at the end of each chapter.

The basic mechanical equations and assumptions are presented in Chap. 1: we review the basic laws for the motion of a particle under Newton's second law, in one and several dimensions, with and without dissipation. Different cases, depending on the acting potential, are presented. We also present two numerical schemes to simulate the corresponding equations of the motion. All this material should be thoroughly used in the sequel as basic building blocks with which to construct methods to solve the proposed problems, ranging from linear algebra to nonlinear programming.

In Chap. 2 we propose a new iterative approach to solve systems of linear equations. The new strategy integrates the algebraic basis of the problem with elements from classical mechanics and the finite difference method. The approach defines two families of convergent iterative methods. Each family is characterized by a linear differential equation, and the methods are obtained from a suitable finite difference scheme to integrate the associated differential equation. These methods are general and depend on neither the matrix dimension nor the matrix structure. We present the basic features of each method. As a consequence, we also present a general method to determine whether a given square matrix is singular or not.

In Chap. 3 we apply the previously developed methods to several examples. We compare these with other similar characteristics, such as Jacobi, Gauss–Seidel, and Steepest Descent Methods and discuss several aspects about choosing the parameter values for the numerical methods.

In Chap. 4 we consider the computation of eigenvectors and eigenvalues of matrices. For a general square matrix, not necessarily symmetric, we construct a family of dynamical systems whose state converges to eigenvectors which correspond to eigenvalues with smallest and biggest real part. We further analyse the convergence and perform several numerical tests. Besides, we extend the application of the method to the effective computation of all eigenvalues with intermediate real part. Some examples and comparisons with the Power Methods are presented in

Chap. 5. We design some ways to enhance the linear convergence of the method, combining it with two different quadratic methods.

In Chaps. 6 and 7 we apply our ideas to solve the so-called programming problems. Chapter 6 is devoted to the classical linear programming problem. We propose a new iterative process to approach the solution of the Primal Problem associated with the linear programming problem: $\max Z = C^{\mathrm{T}}\vec{x}$, with some linear constraints. The method is based on translating the problem to the motion of a Newtonian particle in a constant force field. The optimization of the objective function is related to the search for the minimum of the particle's potential energy. Several solution strategies which depend on the number of dimensions are developed and also illustrated through different examples.

The monograph comes to an end in Chap. 7, which is devoted to the classical quadratic programming: we extend our previous method to the case of optimizing a quadratic objective function with linear constraints as well as to the case of a linear function with quadratic constraints. The method can also be extended to the general case of a nonlinear objective function with linear constraints.

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