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Lecture Notes on O-minimal Structures and Real Analytic Geometry

The Fields Institute for Research
in the Mathematical Sciences

Springer

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Preface

The notes in this volume were produced in conjunction with the Thematic Program in O-minimal Structures and Real Analytic Geometry, held from January to June 2009 at the Fields Institute. Among the activities of our thematic program were three graduate courses, offered to participants and to graduate students from universities in the Greater Toronto Area. Each of these courses was, in turn, split into three modules, and most of these modules were taught by different instructors. Five of the six contributions to this volume arose from the modules taught by the authors: Felipe Cano on the resolution of singularities of vector fields; Chris Miller on o-minimality and Hardy fields; Jean-Philippe Rolin on the construction of o-minimal structures from quasianalytic classes; Fernando Sanz on non-oscillatory trajectories; and Patrick Speissegger on pfaffian sets. The sixth contribution, by Antongiuglio Fornasiero and Tamara Servi, is an adaptation of Wilkie's construction of o-minimal structures from total C^∞ -functions to the nonstandard setting. Their adaptation was carried out concurrently with our program, and the resulting notes fit in naturally with the pfaffian portion of our lectures.

There are only a few dependencies between the contributions: Miller's is used in both Rolin's and Speissegger's, and Rolin's is used in Sanz's. In addition, familiarity with the basics is assumed for o-minimality (van den Dries [4] and Miller and van den Dries [5]) and semianalytic and subanalytic sets (Bierstone and Milman [2]). Further recommended reading are Marker [3] on model theory (basic aspects of which are used in Miller's notes) and Balser [1] on Borel-Laplace summation (used in Sanz's notes).

We thank the Fields Institute for the generous funding provided for our program, and we thank its very competent and helpful staff for making our stay there productive and very enjoyable. Participation of several US-based graduate students and junior postdoctoral researchers was partially funded by NSF Special Meetings Grant DMS-0753096.

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