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Babylonian Mathematical Astronomy: Procedure Texts

Springer

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ISBN 978-1-4614-3781-9 ISBN 978-1-4614-3782-6 (eBook)
DOI 10.1007/978-1-4614-3782-6
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2012935875

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To the memory of John Britton

Preface

This study of the procedure texts of Babylonian mathematical astronomy is aimed at Assyriologists, historians of science, astronomers and others with an interest in Babylonian astronomy. I have organised it in a manner prompted by methodological as well as didactical considerations, in the hope that this will increase its accessibility to readers with different backgrounds. Chapter 1 includes a brief introduction to Babylonian mathematical astronomy, a history of research on this topic and sections devoted to documentary, historical, contextual and methodological aspects. Chapter 2 offers a semantic analysis of the procedures and a systematic investigation of the underlying mathematical concepts. Chapters 3 and 4 are devoted to the algorithms for the planets and the Moon, respectively. While some of the algorithms are undeniably complex, many aspects of the procedure texts can be explored without mathematical or astronomical training. Chapters 2–4 are structured in a bottom-to-top fashion, setting out from elementary concepts (numbers and arithmetic operations) to more complex ones (algorithms and computational systems). Most readers will find little or no difficulty in coming to terms with Chapter 1 and Chapter 2 up to §2.6.7. For those with some elementary mathematical training the rest of Chapter 2 and Chapter 3 should also pose few difficulties. More audacious readers may venture into Chapter 4, which deals with the algorithms for the Moon, by far the most complicated part of Babylonian mathematical astronomy. Critical editions of all known cuneiform tablets and fragments of the corpus can be found in Chapter 5. Many other interesting aspects of Babylonian mathematical astronomy, including its social and institutional context, purpose and applications, empirical foundation, evolution and relationship with astronomical diaries and other observational texts from Babylonia, could not be addressed to any depth in this book. Some of these issues have been or will be addressed in separate publications. An upcoming second volume will deal with the tabular texts of mathematical astronomy. Apart from occasional remarks I also do not address the issue of how accurately the algorithms reproduce the phenomena they aim to predict — for this topic cf. Aaboe (1958) and Swerdlow (1998).

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Berlin
February 14, 2012

Acknowledgements

This book was written in the framework of a research project supported by the Deutsche Forschungsgemeinschaft (DFG) and based at the Institute for Ancient Near Eastern Studies (IANES) of the Eberhard-Karls-Universität Tübingen. It was accepted as a dissertation by the former Fakultät für Kulturwissenschaften of the Eberhard-Karls-Universität Tübingen (2010). My supervisor Konrad Volk (IANES) is gratefully acknowledged for enabling the project and supporting it in all possible ways. The project was prepared while I was a research associate in astrophysics at the Kiepenheuer Institute for Solar Physics (Freiburg i.Br.) and pursuing Assyriological studies at the Orientalisches Seminar of the Albert-Ludwigs-Universität (Freiburg i.Br.). Part of the work was carried out at the Nederlands Instituut voor het Nabije Oosten (NINO) in Leiden during a visiting scholarship (December 2005 – May 2006) provided by the Nederlandse organisatie voor Wetenschappelijk Onderzoek (NWO), for which I thank Wilfred van Soldt (University of Leiden) and Teije de Jong (University of Amsterdam). Eva Cancik-Kirschbaum is acknowledged for her hospitality during numerous stays at the Institut für Altorientalistik of the Freie Universität (Berlin). The manuscript was prepared for publication during a visiting research scholarship at the Institute for the Study of the Ancient World (ISAW), New York University (August 2010 – September 2011). I thank the Trustees of the British Museum, the Vorderasiatisches Museum (Berlin), the Birmingham City Museum, the Oriental Institute (Chicago), the Rare Books Collection of the Columbia University Library (New York), the Metropolitan Museum of Art (New York), the Yale Babylonian Collection (New Haven) and the University of Pennsylvania Museum of Archaeology and Anthropology (Philadelphia) for providing access to their cuneiform collections, and for permission to publish the texts. Nearly all tablets were photographed with a digital camera purchased with financial means provided by the late John Britton. I wish to acknowledge support and input from numerous other individuals, foremost my second supervisor Hermann Hunger, Christopher Walker, Lis Brack-Bernsen, Geert de Breucker, Gregory Chambon, Karine Chemla, Gerd Graßhof, Jens Høyrup, Peter Huber, Alexander Jones, Stefan Maul, Joachim Marzahn, Joachim Oelsner, Christine Proust, Francesca Rochberg, Hein Stadhouders, John Steele, Horst Steible, Marten Stol, Jonathan Taylor and Cornelia Wunsch.

Contents

Abbreviations and symbols

Bibliographical abbreviations

ACT	*Astronomical Cuneiform Texts* (Neugebauer 1955)
ADOG	*Abhandlungen der Deutschen Orient-Gesellschaft*
ADRT	*Astronomical Diaries and Related Texts from Babylonia* (I–III: Sachs & Hunger 1988–1996; V: Hunger & Sachs 2001; VI: Hunger 2006)
AfO	*Archiv für Orientforschung*
AHES	*Archive for History of Exact Sciences*
AHw	*Akkadisches Handwörterbuch*
AIHS	*Archive Internationale d'Histoire des Sciences*
AnOr	*Analecta Orientalia*
AOAT	*Alter Orient und Altes Testament*
AoF	*Altorientalische Forschungen*
AUWE	*Ausgrabungen in Uruk-Warka. Endberichte*
BAK	*Babylonische und assyrische Kolophone* (Hunger 1968)
BaM	*Baghdader Mitteilungen*
BaM Beih	*Baghdader Mitteilungen Beihefte*
BBVO	*Berliner Beiträge zum Vorderen Orient*
BiOr	*Bibliotheca Orientalis*
CDLJ	*Cuneiform Digital Library Journal*
CM	*Cuneiform Monographs*
CT	*Cuneiform Texts from Babylonian Tablets in the British Museum*
CTMMA	*Cuneiform Texts from the Metropolitan Museum of Art*
DIO	*The International Journal of Scientific History*
FAOS	*Freiburger Altorientalische Studien*
Fs Hunger	*Festschrift Hunger* (*WZKM* 97, 2007)
Fs Oelsner	*Festschrift Oelsner* (Marzahn & Neumann 2000)
Fs Slotsky	*Festschrift Slotsky* (Ross 2008)
GAG	*Grundriß der akkadischen Grammatik* (von Soden 1969)
HAMA	*History of Ancient Mathematical Astronomy* (Neugebauer 1975)
JAOS	*Journal of the American Oriental Society*
JCS	*Journal of Cuneiform Studies*

JHA	*Journal for the History of Astronomy*
JNES	*Journal of Near Eastern Studies*
KDVSMM	*Kongelige Danske Videnskabernes Selskab Matematisk-fysiske Meddelelser*
LBAT	*Late Babylonian Astronomical Texts* (Pinches & Sachs 1955)
MCT	*Mathematical Cuneiform Texts* (Neugebauer & Sachs 1986)
MKT	*Mathematische Keilschrifttexte* (Neugebauer 1935–1937)
NABU	*Nouvelles Assyriologiques Brèves et Utilitaires*
OECT	*Oxford Editions of Cuneiform Texts*
PAPS	*Proceedings of the American Philosophical Society*
QS	*Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*
RA	*Revue d'Assyriologie et d'Archéologie Orientale*
RlA	*Reallexikon der Assyriologie und Vorderasiatischen Archäologie*
SpTU	*Spätbabylonische Texte aus Uruk*
SSB	*Sternkunde und Sterndienst in Babel* (I: Kugler 1907; II: Kugler 1909, 1912–1924)
TAPS	*Transactions of the American Philosophical Society*
TEBR	*Textes babyloniens d'époque récente* (Durand 1981)
TMS	*Textes Mathématiques de Suse* (Bruins & Rutten 1961)
TU	*Tablettes d'Uruk* (Thureau-Dangin 1922)
UOS	*Under One Sky. Astronomy and Mathematics in the Ancient Near East* (Steele & Imhausen 2002)
UVB	*Uruk, vorläufige Berichte*
WZKM	*Wiener Zeitschrift für die Kunde des Morgenlandes*
ZA	*Zeitschrift für Assyriologie*

Assyriological abbreviations

A	tablet siglum of the Oriental Institute museum (Chicago)
Akk.	Akkadian
AO	tablet siglum of the Louvre (Paris)
AT	auxiliary table
acc.	accusative
adv.	adverb(ial)
BCM	tablet siglum of the Birmingham City Museum
BE	tablet siglum of the Koldewey excavations at Babylon
BM	tablet siglum of the British Museum (London)
CBS	tablet siglum of the University of Pennsylvania Museum of Archaeology and Anthropology (Philadelphia)
c.	common (gender)
col.	colophon(s)
conj.	conjunction
DN	name of a deity
DT	1. daily motion table
	2. tablet siglum of a collection in the British Museum (London)
E	edge

EAE	*Enūma Anu Enlil*
ED	Early Dynastic (2900–2350 BC)
f.	feminine
gen.	genitive
inf.	infinitive
inv.	invocation
LB	Late Babylonian (539 BC–100 AD)
LE	lower edge
lit.	literally
loc.	locative
MB	Middle Babylonian (1500–1100 BC)
MLC	tablet siglum of the Morgan Library Collection (Yale)
MMA	tablet siglum of the Metropolitan Museum of Art (New York)
MN	month name
Ms.	manuscript
m.	masculine
NA	Neo Assyrian (900–612 BC)
NB	Neo Babylonian (750–539 BC)
NMAT	non-mathematical astronomical text(s)
nom.	nominative
O	obverse
OB	Old Babylonian (1800–1600 BC)
P	procedure
PN	personal name
PT	procedure text
p.	person
pl.	plural
poss. suff.	possessive suffix
pres.	present tense
pret.	preterite tense
ptc.	participle
QN	name of a quantity
R	reverse
Rm	tablet siglum of the Rassam collections in the British Museum (London)
SN	name of a synodic phenomenon
Sp	tablet siglum of the Spartali collections in the British Museum (London)
ST	synodic table
Sum.	Sumerian
sg.	singular
stat.	stative
T	table
TT	template table

U	tablet siglum of the Uruk collection in the Eski Şark Eserleri Müzesi (Istanbul)
UE	upper edge
VAT	tablet siglum of the Vorderasiatisches Museum (Berlin)
verb. adj.	verbal adjective
W	tablet siglum of the German excavations at Uruk

Astronomical abbreviations and symbols

Unless stated otherwise the conventions are the same as in *ACT* and *HAMA*.

A	system A	
A	synodic arc of the Moon and the Sun (Moon system B)	
AR	Acronychal Rising (outer planets)	[*ACT*, *HAMA*: Θ]
B	system B	
B	zodiacal position (lunar and planetary systems)	
b_j	'trailing' boundary of zone j of step function (type-A systems)	
$^{\mathrm{b}}$	*bēru*, 'mile' (unit of arc and time)	
C	system C (Venus)	
C	duration of daylight (Moon systems A, B, K)	
C'	correction to the time of the lunation (Moon system A)	
CO	Conjunction	
c_j	transition coefficient for step function in zone j (type-A systems)	
c'_j	transition coefficient for generalised step function in zone j (type-A systems)	
c, c_k, c'_k	interpolation coefficient (for interval k)	
c_Ψ	maximum value of eclipse magnitude (parameter of Ψ, Ψ' and Ψ'' in Moon systems A, B)	
c_τ	constant difference between synodic time τ and synodic arc σ (planetary systems)	[*ACT*, *HAMA*: c]
c'_τ	constant difference between synodic time τ and total synodic arc Σ (planetary systems)	
$c'_{\delta\tau}$	constant difference between temporal push $\delta\tau$ and angular push $\delta\Sigma$ (Mars system A)	
$^{\mathrm{c}}$	cubit (unit of arc)	
D	duration of the night (Moon systems A, B)	[*ACT*, *HAMA*: D']
D'	half the duration of the night (Moon system B)	[*ACT*, *HAMA*: D]
d	difference of a function for 1 synodic cycle [$= df(1,0)$]	
$dB(s)$	or $dB(s,t)$: net displacement for s synodic cycles, where t is nearest whole number of revolutions	
$dB_j(s)$	or $dB_j(s,t)$: same, for zone j (type-A systems)	
$df(s)$	or $df(s,t)$: net difference of f for s synodic cycles, where t is nearest whole number of oscillations of f	
$df_j(s)$	or $df_j(s,t)$: same, for zone j (type-A systems)	
d	(whole) day number [1–30] (Moon systems A, B, K)	
$^{\mathrm{d}}$	day	
E	distance to the ecliptic (Moon systems A, B, K; Jupiter; Saturn)	
EF	Evening First (inner planets)	[*ACT*, *HAMA*: Ξ]
EL	Evening Last (inner planets)	[*ACT*, *HAMA*: Ω]
ES	Evening Station (inner planets)	[*ACT*, *HAMA*: Ψ]
e	yearly epact (length of the year in excess of 12^{m})	

F	the Moon's daily displacement along the zodiac measured in degrees per day (Moon systems A, B)	
F'	idem, measured in degrees per time degree	
FA	First Appearance (outer planets)	[*ACT*, *HAMA*: Γ]
FM, fm	Full Moon (opposition)	[*ACT*, *HAMA*: index 2]
f	unspecified function	
$^{\mathrm{f}}$	finger (unit of arc)	
G	duration of synodic month -29^{d} (Moon systems A, B, K)	
GI_6	Lunar Six interval near Full Moon: sunset to first moonrise after sunset (Moon systems A, B, K)	
GY	Goal Year	
H	monthly difference of J (Moon system B)	
h	astronomical altitude (height above horizon)	
IC	Inferior Conjunction	
i	index for the event number of a synodic phenomenon, lunation number, day number or tithi number	
J	zodiacal correction to G (Moon systems A, B)	
j	index for the zone of a step function	
K	system K (Moon)	
K	time interval between successive lunations -29^{d} (Moon systems A, B)	
KUR	Lunar Six interval near New Moon: last visible moonrise before sunrise to sunrise (Moon systems A, B, K)	
k	index for the control values of an interpolation scheme	
L	time of lunation with respect to preceding midnight (Moon system B)	
LA	Last Appearance (outer planet)	[*ACT*, *HAMA*: Ω]
M	1. maximum of a function	
	2. time between lunation and sunset or sunrise (Moon systems A, B, K)	
ME	Lunar Six interval near Full Moon: last moonrise before sunset to sunset (Moon systems A, B, K)	
MF	Morning First (inner planets)	[*ACT*, *HAMA*: Γ]
ML	Morning Last (inner planets)	[*ACT*, *HAMA*: Σ]
MS	Morning Station (inner planets)	[*ACT*, *HAMA*: Φ]
m	1. minimum of a function	
	2. number of additional full revolutions of the Sun for every synodic cycle	
$^{\mathrm{m}}$	synodic month (unit of the event frame)	
$\overline{\mathrm{m}}$	mean synodic month (unit of time)	
N	1. time interval between lunation and Lunar Six event (Moon systems A, B)	
	2. number of elementary steps contained in total synodic arc Σ (type-A systems)	
NA	Lunar Six interval near Full Moon: sunrise to first moonset after sunrise (Moon systems A, B, K)	
NA_1	Lunar Six interval near New Moon: sunset to first visible moonset after sunset (Moon systems A, B, K)	
NM, nm	New Moon (conjunction)	[*ACT*, *HAMA*: index 1]
n	whole number of additional revolutions of the planet per synodic cycle	
$^{\mathrm{nin}}$	*nindanu*, 'rod' (unit of arc and time)	
O	corrected elongation of Moon from Sun or anti-Sun (Moon system B)	
OP	Opposition (Moon, outer planets)	
P	period measured in fractional synodic events after which a function returns to the same value	

p	1. period, measured in synodic events, of more rapidly varying function underlying tabulated function	
	2. index for different pushes within 1 synodic cycle (planetary systems)	
Q	1. contribution to Lunar Six from distance along the ecliptic (Moon systems A, B)	
	2. unspecified quantity	
QN	name of an unspecified quantity	
q	ratio between rising or setting time and distance along the zodiac (Moon systems A, B)	
R	1. contribution to Lunar Six from the Moon's distance to the ecliptic (Moon systems A, B)	
	2. refraction angle	
RA	right ascension (= α)	
r	ratio between rising or setting time and distance to the ecliptic (Moon systems A, B)	
r_j	transition coefficient for step function in zone j	
S_1, S_2...	schemes for the subdivision of the synodic cycle	
S1	First Station (outer planet)	[*ACT*, *HAMA*: Φ]
S2	Second Station (outer planet)	[*ACT*, *HAMA*: Ψ]
SC	Superior Conjunction (inner planet)	
SIGN	name of unspecified zodiacal sign	
SN	name of an unspecified synodic phenomenon	
s	whole number of synodic events	
$\check{S}U_2$	Lunar Six interval near Full Moon: last moonset before sunrise to sunrise (Moon systems A, B, K)	
T	1. column containing year number and month name (Moon systems A, B)	
	2. time (planetary systems)	
T_{rise}	time interval between rising of planet at FA and sunrise (Jupiter systems A, A"')	
T_{set}	time interval between sunset and setting of planet at LA (Jupiter systems A, A"')	
t	nearest whole number of oscillations of a function for a close return to the same value	
UT	Universal Time	
v	'daily' displacement along the zodiac	
W	duration of 6 synodic months modulo a whole number of days (Moon system A)	
w	generalised step function (Moon system A)	
w_j	preliminary value of generalised step function in zone j (Moon system A)	
X	unidentified computational system	
X_m	lunar variation of the Moon's net displacement for 1^m? (Moon system A)	
X_s	lunar variation of the Moon's net displacement for 223^m? (Moon system A)	
Y	1. whole number of years after which a function returns to exactly the same value	
	2. zodiacal correction to Λ (Moon system A)	
y	whole number of years after which a function returns to nearly the same value ('close return')	
Z	1. whole number of oscillations after which a function returns to exactly the same value	
	2. zodiacal correction to W (Moon system A)	
zod.	zodiacal	
α	right ascension	
α_j	length of zone j of step function	
β	distance to the ecliptic [degrees] (Moon systems A, B)	
β_0	distance to the ecliptic at lunation [degrees] (Moon systems A, B)	

Δ	1. amplitude of a function ($= M - m$)	
	2. disk correction (Moon system A)	
ΔB	zodiacal displacement (other than synodic arc σ, push $\delta\Sigma$, or net displacement dB)	
δ	1. declination	
	2. elementary step (of zigzag function)	
δ_j	elementary step of step function	[*HAMA*: I_j]
δN_p	number of elementary steps contained in angular push p (type-A systems)	[*HAMA*: s_p]
$\delta\Sigma$	angular push (subdivision of Σ)	[*ACT*: Δ; *HAMA*: $\delta\lambda$]
$\delta\Sigma_p$	angular push p	
$\delta\Sigma_{pj}$	angular push p in zone j (type-A systems)	
$\delta\tau$	temporal push (subdivision of τ)	
$\delta\tau_p$	temporal push p	
η	elongation	
ε	1. obliquity of the ecliptic	
	2. correction to the lunar elongation (step 3 of Lunar Six module of Moon system B)	
θ	sidereal time	
κ	half width of nodal region (Moon system A)	
Λ	duration of 12 synodic months -354^{d} (Moon system A)	
λ	ecliptical longitude	
μ	mean value of a function	
ν_j	number of elementary steps contained in zone j (step function)	[*HAMA*: ν_j, π_j]
Π	number period (smallest whole number of events after which a function returns to exactly the same value)	
Φ	duration of 223 synodic months -6585^{d} (Moon system A)	
ϕ	1. geographical latitude	
	2. basic interval between adjacent Φ_k in the interpolation schemes for G, W and Λ (Moon system A)	
Ψ, Ψ', Ψ''	eclipse magnitude and its variants (Moon systems A, B)	
Σ	total synodic arc	[*ACT*: $\Delta\Lambda$]
σ	synodic arc	[*ACT*: $\Delta\lambda$]
σ_j	preliminary value of step function for synodic arc in zone j (type-A systems)	[*ACT*, *HAMA*: w_j]
τ	1. synodic time	[*ACT*, *HAMA*: $\delta\tau$]
	2. hour angle	
τ_j	synodic time in zone j (planetary systems of type A, C)	
$^{\tau}$	(real) tithi (unit of the event frame)	
$^{\overline{\tau}}$	mean tithi (unit of time)	[*ACT*, *HAMA*: $^{\tau}$]

Names of the planets, zodiacal signs, months, regnal years and units

names of the planets			
transliteration	Akkadian (literal translation)	translation	symbol
[^{d}]*sin*(30)	*Sîn*	Moon	☽
šamaš$_2$(20), ^{d}**utu**	*Šamaš*	Sun	☉
[^{d}]**gu$_4$.ud**	*Šiḫṭu* (‘Attack; Jump’)	Mercury	☿
[^{d}]*dil-bat*	*Dilbat*	Venus	♀
AN	*Ṣalbatānu*	Mars	♂
mul$_2$.babbar	*Peṣû* (‘The white one’)	Jupiter	♃
genna(TUR.DIŠ)	*Kajjamānu* (‘The steady one’)	Saturn	♄

Occasionally used alternative names: *Sagmegar*(**sag.me.gar**) = Jupiter, *Kajjamānu*(**sag.uš**) = Saturn.

signs of the zodiac						
transliteration	older form	Akkadian	literal translation	sign	abbr.	symbol
ḫun, **lu**	$^{mul.lu_2}$**ḫun.ga$_2$**	*agru*	hireling	Aries	Ari	♈
mul$_2$[**.mul$_2$**]	**mul.mul**	*zappu*	stars (Sum.); bristle (Akk.)	Taurus	Tau	♉
maš[**.maš**]	^{mul}**maš.tab.ba**	*māšu, tū’amū*	twins	Gemini	Gem	♊
alla	^{mul}**al.lul**	*alluttu*	crab	Cancer	Cnc	♋
a	^{mul}**ur.a**	*nēšu*	lion	Leo	Leo	♌
absin, **absin$_0$**(KI)	^{mul}**ab.sin$_2$**	*šer’u*	furrow	Virgo	Vir	♍
rin$_2$	^{mul}**giš.rin$_2$**	*zibānītu*	scales	Libra	Lib	♎
gir$_2$[**.tab**]	^{mul}**gir$_2$.tab**	*zuqaqīpu*	scorpion	Scorpius	Sco	♏
pa	^{mul}**pa.bil.sag**	*Pabilsag*	Pabilsag	Sagittarius	Sgr	♐
maš$_2$	^{mul}**suḫur.maš$_2$**	*suḫurmāšu*	goat-fish	Capricorn	Cap	♑
gu	^{mul}**gu.la**	*Gula*	Gula	Aquarius	Aqr	♒
zib[**.me**]	^{mul}**kun.meš**	*zibbātu*	tails	Pisces	Psc	♓

Also listed are the official abbreviations which are used in all translations.

month names			
transliteration	OB Nippur	Akkadian (+literal translation)	translation
bar	^{iti}**bara$_2$.zag.gar**	*Nisannu*	I
gu$_4$	^{iti}**gu$_4$.si.su$_3$**	*Ajjaru*	II
sig	^{iti}**sig$_4$.^{giš}u$_5$.šub.ba.ga$_2$.gar**	*Simānu*	III
šu	^{iti}**šu.numun**	*Dûzu*	IV
ne	^{iti}**ne**.IZI.**gar**	*Abu*	V
kin	^{iti}**kin.^{d}inana**	*Ulūlu*	VI
kin.2[**.kam**]		*Ulūlu arkû* (‘second Ulūlu’)	VI$_2$
du$_6$	^{iti}**du$_6$.ku$_3$**	*Tašrītu*	VII
apin	^{iti}**apin.du$_8$.a**	*Araḫsamna*	VIII
gan	^{iti}**gan.gan.e$_3$**	*Kislīmu*	IX
ab	^{iti}**ab.e$_3$**	*Ṭebētu*	X
ud$_2$(ZIZ$_2$)	^{iti}**ud$_2$.duru$_5$**	*Šabāṭu*	XI
še	^{iti}**še.kin.ku$_5$**	*Addaru*	XII
diri[**.še**]	^{iti}**diri.še.kin.ku$_5$**	*Addaru arkû* (‘second Addaru’)	XII$_2$

All month names are translated with Roman numbers. The cuneiform sign for month XI is transliterated as ZIZ$_2$ even though the month names of OB Nippur (Cohen 1993) imply that the proper reading is **ud$_2$**.

regnal years		
ruler	regnal years	Julian yr (BC)
Xerxes	1–21	485/4
Artaxerxes I	1–41	464/3
Darius II	1–19	423/2
Artaxerxes II	1–46	404/3
Artaxerxes III	1–21	358/7
Arses	1–2	337/6
Darius III	1–5	335/4
Alexander the Great	7–14	330/329
Philip Arrhidaeus	2–8	322/1
Alexander IV	2–5	315/4
Seleucid Era (SE)	1–	311/0
Arsacid Era (AE)	1–	247/6

Late Achaemenid and early Seleucid rulers. The third column contains the Julian equivalent of the first regnal year.

units of measurement						
transliteration	Akkadian	translation	symbol	internal equivalence	modern equivalence	
					angular	temporal
danna	*bēru*	'mile'	^{b}	$1^{b} = 30°$	30°	2 hrs.
UŠ	?	(time) degree	$^{\circ}$	$1° = 60^{nin}$	1°	4 min.
ninda(GAR)	*nindanu* (?)	'rod' (?)	^{nin}		0;1° = 1' (arcmin)	4 sec.
kuš$_3$	*ammatu*	cubit	^{c}	$1^{c} = 24^{f} = 2°$	2°	
[šu].si, *u*	*ubānu*	finger	^{f}	$1^{f} = 6^{še}$	0;5°	
še	*uṣṣatu*	barleycorn	^{še}		0;0,50°	

Top: the 'degree system', used for angles and time intervals. Bottom: the 'cubit system', used for angular distance to the ecliptic and eclipse magnitude.

Factor diagram for the 'degree system':

$$1 \text{ 'mile' } (^{b}) \xleftarrow{30} 1 \text{ degree } (^{\circ}) \xleftarrow{60} 1 \text{ 'rod' } (^{nin})$$

Factor diagram for the 'cubit system':

$$1 \text{ cubit } (^{c}) \xleftarrow{24} 1 \text{ finger } (^{f}) \xleftarrow{6} 1 \text{ barleycorn } (^{še})$$