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Operator Inequalities of Ostrowski and Trapezoidal Type

Springer

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To my beloved friend and wife Nicoleta
for our 30th anniversary

Preface

Linear Operator Theory in Hilbert spaces plays a central role in contemporary mathematics with numerous applications for Partial Differential Equations, in Approximation Theory, Optimization Theory, Numerical Analysis, Probability Theory and Statistics, and other fields.

The aim of this short book is to present recent results concerning Ostrowski and Trapezoidal type inequalities for continuous functions of bounded selfadjoint operators on complex Hilbert spaces.

The book is intended for use by both researchers in various fields of Linear Operator Theory and Mathematical Inequalities, domains which have grown exponentially in the last decade, as well as by postgraduate students and scientists applying inequalities in their specific areas.

In the first chapter we recall some fundamental facts concerning bounded selfadjoint operators on complex Hilbert spaces. The generalized Schwarz's inequality for positive selfadjoint operators as well as some results for the spectrum of this class of operators are presented. Then, we introduce and explore the fundamental results for polynomials in a linear operator, the continuous functions of selfadjoint operators, and the step functions of selfadjoint operators. By the use of these results we then introduce the spectral decomposition of selfadjoint operators (the *Spectral Representation Theorem*) that will play a central role in the rest of the book. This result is used as a key tool in obtaining various new inequalities for continuous functions of selfadjoint operators, functions which are of bounded variation, Lipschitzian, monotonic or absolutely continuous. Another tool that will greatly simplify the error bounds provided in the book is the *Total Variation Schwarz's Inequality* for which a simple proof is offered.

The next chapter is devoted to the Ostrowski's type inequalities. They provide sharp error estimates in approximating the value of a function by its integral mean and can be utilized to obtain a priory error bounds for different quadrature rules in approximating the Riemann integral by different Riemann sums. They also shows, in general, that the mid-point rule provides the best approximation in the class of all Riemann sums sampled in the interior points of a given partition.

As revealed by a simple search in *MathSciNet* with the key words "Ostrowski" and "inequality" in the title, an exponential evolution of research papers devoted to this result has been registered in the last decade. There are now at least 280 papers that can be found by performing the above search. Numerous extensions, generalizations in both the integral and discrete case have been discovered. More general versions for n-time differentiable functions, the corresponding versions on time scales, for vector valued functions or multiple integrals have been established as well. Numerous applications in Numerical Analysis, Probability Theory, and other fields have been also given.

In this chapter we present some recent results obtained by the author in extending Ostrowski inequality in various directions for continuous functions of selfadjoint operators in complex Hilbert spaces. Applications for mid-point inequalities and some elementary functions of operators are provided as well.

From a complementary viewpoint to Ostrowski/mid-point inequalities, trapezoidal type inequality provide a priory error bounds in approximating the Riemann integral by a (generalized) trapezoidal formula.

Just like in the case of Ostrowski's inequality the development of these kind of results have registered a sharp growth in the last decade with more than 50 papers published, as one can easily asses this by performing a search with the key word "trapezoid" and "inequality" in the title of the papers reviewed by *MathSciNet*.

Numerous extensions, generalizations in both the integral and discrete case have been discovered. More general versions for n-time differentiable functions, the corresponding versions on time scales, for vector valued functions or multiple integrals have been established as well.

In the third chapter we present some recent results obtained by the author in extending trapezoidal type inequality in various directions for continuous functions of selfadjoint operators in complex Hilbert spaces.

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