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P. Dazord A. Weinstein
Editors

Symplectic Geometry, Groupoids, and Integrable Systems

Séminaire Sud Rhodanien de Géométrie à Berkeley (1989)

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Preface

The papers in this volume are based on lectures given during the meeting of the Séminaire Sud Rhodanien de Géométrie which we organized at MSRI from May 22 to June 2, 1989, as part of a year-long program on Symplectic Geometry and Mechanics.

The Séminaire Sud Rhodanien de Géométrie (SSRG) was established in 1982 by geometers and mathematical physicists at the Universities of Avignon, Lyon, Marseille, and Montpellier, with the aim of developing and coordinating research in symplectic geometry and its applications to analysis and mathematical physics. It has been designated by the Centre Nationale de la Recherche Scientifique as a "Groupement de Recherche" (G.D.R. 144), centered at the Université Claude Bernard (Lyon I). From the beginning, the SSRG has involved the cooperation of colleagues from other universities inside and outside France; in addition to the editors of this volume, its Scientific Committee consists of D. Bennequin, P. Libermann, A. Lichnerowicz, C.-M. Marle, J.-M. Morvan, P. Molino, and J.-M. Souriau. In particular, there have always been strong connections with the University of California at Berkeley, making this other "UCB" into a virtual fifth pole of the SSRG.

Through its international meetings, of which the first five were held at Lyon, Montpellier, and Marseille, the SSRG has become an important center of exchange for the latest developments in symplectic geometry and its applications. It seemed natural, therefore, to have this sixth meeting at MSRI in Berkeley in conjunction with the "symplectic year" 1988-89. Roughly half of the speakers came from France, while the others were resident participants in the longer MSRI program.

Among the subjects discussed at the meeting, a special role was given to the theory of "symplectic groupoids," which has been since 1987 the subject of a fruitful collaboration involving geometers from Berkeley, Lyon, and Montpellier. It may be useful to the reader to have a brief introduction to this subject, as a guide to the detailed articles contained in this volume.

Symplectic groupoids are the global objects whose relation to Poisson manifolds is the same is that of Lie groups to Lie algebras. They were introduced independently by M. Karasev and one of us (A.W.). An expository paper written by P.D. and A.W. in collaboration with A. Coste can be found in the Publications of the Department of Mathematics of Lyon (2/A 1987).

A symplectic groupoid is a symplectic manifold with a compatible partially defined multiplication which satisfies the axioms of a category in which all morphisms are invertible. The set of objects (also called the base) for this category inherits a Poisson structure for which the source and target maps are Poisson and anti-Poisson respectively. For example, the cotangent bundle of a Lie group G has a structure of symplectic groupoid for which the base is the dual Lie algebra $\mathfrak{g}^*$ with its Lie-Poisson structure.

The following problem is fundamental: which Poisson manifolds are integrable in the sense that they occur as the bases of symplectic groupoids? According to Lie's third theorem and the example in the preceding paragraph, every (finite dimensional) $\mathfrak{g}^*$ with a Lie-Poisson structure is integrable. There exists for every Poisson manifold a local symplectic groupoid (Karasev, A.W.), but there are non-trivial obstructions to global integrability, even for regular Poisson manifolds (P.D.).

Symplectic groupoids play a special role in the theory of quantization, where they serve as geometric models for "quantum" algebras. More precisely, one may attempt to construct noncommutative algebras from Poisson manifolds by the following series of steps: 1. find, if it exists, a symplectic groupoid for the symplectic manifold; 2. prequantize the symplectic groupoid to get a bundle whose sections carry a natural multiplication; 3. find a polarization which is sufficiently compatible with the groupoid structure so that the algebra in Step 2 can be cut down to one which is roughly the "size" of the functions on the original Poisson manifold. This program applies, for example to the dual of a Lie algebra, where it produces the convolution algebra of the corresponding Lie group. This example may even be seen as a successful case in a general program to construct quantum groups by quantization of the symplectic groupoids of Poisson Lie groups.

Several of the papers in this volume are directly concerned with symplectic groupoids. Albert and Dazord give a new development of the basic theory of symplectic groupoids, based on the idea of a pseudogroup of left or right translations. It can be read as an introduction to the subject. Dazord and Hector show that the integration problem can always be solved for a class of regular Poisson manifolds called totally aspherical, on which the cohomological obstruction to integrability vanishes and the foliation by symplectic leaves has no vanishing cycles. Lashof gives an algebraic topological proof of a result of A.W. on the quantization of fundamental groupoids of symplectic manifolds. In the paper of Weinstein, the quantization program is carried out in full for the case of translation-invariant Poisson structures on tori, where it is shown to lead to the algebras which are already known under the name of "noncommutative tori". Xu's paper is devoted to a notion of Morita equivalence for symplectic manifolds which is the geometric analog of a fundamental equivalence relation in the theory of noncommutative algebras.

Other papers are connected with Poisson manifolds without explicit reference to groupoids. Dazord and Sondaz consider a class of Poisson structures on groups which include the multiplicative structures (Poisson Lie groups) as well as the left and right invariant structures. The latter are the subject of the work of Boyom and of Medina and Revoy. Lu's paper is closely related, giving an extension of the standard theories of momentum mappings and reduction to Poisson actions of Poisson Lie groups. Dufour's paper discusses an interesting case of the linearization problem—i.e. the problem of determining when a class of Poisson structures are locally isomorphic to those of Lie-Poisson type. Marle develops some general properties of Jacobi manifolds, a class of objects which contains both Poisson (hence symplectic) and contact manifolds.

The symplectic leaves of a Poisson manifold form a singular foliation. Suzuki is concerned with the construction of C^* algebras for more general singular foliations—both the algebras and the foliations bear a close relation with the geometry of Poisson manifolds. Stefan foliations, also called singular foliations, are generated as well by the integral manifolds of completely integrable systems. Boucetta generalizes to the singular case some results of Duistermaat on fibrations by compact lagrangian submanifolds. Desolneux-Moulis describes the bifurcations of lagrangian Stefan foliations in the neighborhood of a transversely hyperbolic invariant torus, a result which Koiller uses to obtain intrinsic Melnikov formulas. Dufour and Molino extend to actions of $\mathbb{R}^n$ with a compact orbit a theorem of Eliasson on action-angle variables with singularities.

Finally, the volume contains several papers on assorted aspects of symplectic manifolds, their geometry, topology, and quantization. Dazord and Patissier establish the existence of an obstruction, postulated by Karasev and Maslov and related to the first Chern class, to asymptotic quantization. Eliashberg and Ratiu prove that the group of symplectomorphisms of the unit ball in $\mathbb{R}^{2n}$ has infinite diameter. Gotay and Tuynman prove that any hamiltonian action of a compact connected Lie group arises by reduction from a linear hamiltonian action on $\mathbb{R}^{2n}$. Lafontaine uses Gromov's theory to obtain a pseudo-holomorphic version of a theorem of Hadamard.

The meeting SSRG-MSRI could not have taken place without the generous assistance of several organizations. In particular, we wish to thank:

- I. Kaplansky of the MSRI, which provided a substantial part of the funding for the French visitors, and whose staff assured the efficient running of the meeting and hospitality for visitors.
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Finally, we wish to thank Margaret Pattison at MSRI for putting the papers in this volume into final TEX form.

Pierre Dazord
Alan Weinstein
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