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K.T. Smith

Power Series from a Computational Point of View

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PREFACE

At the end of the typical one quarter course on power series the students lack the means to decide whether $1/(1+x^2)$ has an expansion around any point $\neq 0$, or the tangent has an expansion anywhere — and the means to evaluate and predict errors.

In using power series for computation the main problems are: 1) To predict a priori the number N of terms needed to do the computation with a specified accuracy; and 2) To find the coefficients $a_0,\ldots,a_N$. These are the problems addressed in the book.

Typical computations envisioned are: calculate with error $\leq 10^{-6}$ the integrals

$$\int_0^{\pi/2} (\pi/2-x)\tan x \, dx \quad , \qquad \int_0^1 x(1+e^x)^{1/2} \, dx,$$

or the solution to the differential equation

$$y''+(\sin x)y'+x^2 y = 0, \quad y(0) = 0, \; y'(0) = 1,$$

on the interval $0 \leq x \leq 1$.

This computational point of view may seem narrow, but, in fact, such computations require the understanding and use of many of the important theorems of elementary analytic function theory: Cauchy's Integral Theorem, Cauchy's Inequalities, Unique Continuation, Analytic Continuation and the Monodromy Theorem, etc. The computations provide an effective motivation for learning the theorems and a sound basis for understanding them. To other scientists the rationale for the computational point of view might be the need for efficient accurate calculation; to mathematicians it is the motivation for learning theorems and the practice with inequalities, ϵ's, δ's, and N's.

Throughout the book $\epsilon = 10^{-6}$. Experience shows that 10^{-6} (or any other specific small number) is more acceptable and challenging to students than a vague and mysterious ϵ, while, of course, there is no difference in the mathematical analysis. 10^{-6} is chosen so that those who want to can perform realistic computations on a 16 bit microcomputer. The computer code is usually a mathematical proof in a disguise that is appealing to students, and it is strongly recommended as a required part of the problem solutions, simply as a learning device.

Since the book contains complete proofs of the theorems cited above, it is clear that the whole cannot be covered in one quarter. A one quarter course, especially one for engineers, physicists, etc., might cover Chapters 1 and 2 with intensive discussion of the meaning and application of the theorems, but without proofs. (This has been done several times with gratifying success.) A two or three quarter course might cover the whole with proofs and other topics. (The simple proof of the general homotopy version of Cauchy's Theorem was devised in such a course about twenty-five years ago.)

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