Undergraduate Texts in Mathematics

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Intermediate Real Analysis

With 100 Illustrations

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To
Wilhelm Magnus

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Preface

There are a great deal of books on introductory analysis in print today, many written by mathematicians of the first rank. The publication of another such book therefore warrants a defense. I have taught analysis for many years and have used a variety of texts during this time. These books were of excellent quality mathematically but did not satisfy the needs of the students I was teaching. They were written for mathematicians but not for those who were first aspiring to attain that status. The desire to fill this gap gave rise to the writing of this book.

This book is intended to serve as a text for an introductory course in analysis. Its readers will most likely be mathematics, science, or engineering majors undertaking the last quarter of their undergraduate education. The aim of a first course in analysis is to provide the student with a sound foundation for analysis, to familiarize him with the kind of careful thinking used in advanced mathematics, and to provide him with tools for further work in it. The typical student we are dealing with has completed a three-semester calculus course and possibly an introductory course in differential equations. He may even have been exposed to a semester or two of modern algebra. All this time his training has most likely been intuitive with heuristics taking the place of proof. This may have been appropriate for that stage of his development. However, once he enters the analysis course he is subject to an abrupt change in the point of view and finds that much more is demanded of him in the way of rigorous and sound deductive thinking. In writing the book we have this student in mind. It is intended to ease him into his next, more mature stage of mathematical development.

Throughout the text we adhere to the spirit of careful reasoning and rigor that the course demands. We deal with the problem of student adjustment to the stricter standards of rigor demanded by slowing down the pace at which topics are covered and by providing much more detail in the proofs than is customary in most texts. Secondly, although the book contains its share of abstract and general results, it concentrates on the specific and concrete by applying these theorems to gain information about some of the important functions of analysis. Students are often presented and even have proved for them theorems of great theoretical significance without being given the opportunity of seeing them "in action" and applied in a nontrivial way. In our opinion, good pedagogy in mathematics should give substance to abstract and general results by demonstrating their power.

This book is concerned with real-valued functions of one real variable. There is a chapter on complex numbers, but these play a secondary role in the development of the material, since they are used mainly as computational aids to obtain results about trigonometric sums.

For pedagogical reasons we avoid "slick" proofs and sacrifice brevity for straightforwardness.

The material is developed deductively from axioms for the real numbers. The book is self-contained except for some theorems in finite sets (stated without proof in Chapter II) and the last theorem in Chapter XIV. In the main, any geometry that is included is there for purposes of visualization and illustration and is not part of the development. Very little is required from the reader in the way of background. However, we hope that he has the desire and ability to follow a deductive argument and is not afraid of elementary algebraic manipulation. In short, we would like the reader to possess some "mathematical maturity." The book's aim is to obtain all its results as logical consequences of the fifteen axioms for the real numbers listed in Chapter I.

The material is presented sequentially in "theorem–proof–theorem" fashion and is interspersed with definitions, examples, remarks, and problems. Even if the reader does not solve all the problems, we expect him to read each one and to understand the result contained in it. In many cases the results cited in the problems are used as proofs of later theorems and constitute part of the development. When the reader is asked, in a problem, to prove a result which is used later, this usually involves paralleling work already done in the text.

Chapters are denoted by Roman numerals and are separated into sections. Results are referred to by labeling them with the chapter, section, and the order in which they appear in the section. For example, Theorem X.6.2 refers to the second theorem of section 2 in Chapter X. When referring to a result in the same chapter, the Roman numeral indicating the chapter is omitted. Thus, in Chapter X, Theorem X.6.2 is referred to as Theorem 6.2.

We also mention a notational matter. The open interval with left endpoint a and right endpoint b is written in the book as $(a;b)$ using a semi-colon between a and b, rather than as (a, b). The latter symbol is reserved for the ordered pair consisting of a and b and we wish to avoid confusion.

I owe a special debt of gratitude to my friend and former colleague Professor Abe Shenitzer of York University in Ontario, Canada, for patiently reading through the manuscript and editing it for readability.

My son Joseph also deserves special thanks for reading most of the material, pointing out errors where he saw them, and making some valuable suggestions.

Thanks are due to Professors Eugene Levine and Ida Sussman, colleagues of mine at Adelphi University, and Professor Gerson Sparer of Pratt Institute, for reading different versions of the manuscript.

Ms. Maie Croner typed almost all of the manuscript. Her skill and accuracy made the task of readying it for publication almost easy.

I am grateful to the staff at Springer-Verlag for their conscientious and careful production of the book.

To my wife Sylvia I give thanks for her patience through all the years the book was in preparation. תושלב"ע

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