

**Springer Series in Cognitive Development**

*Series Editor*  
Charles J. Brainerd

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Children's Logical and Mathematical Cognition:  
Progress in Cognitive Development Research  
*Charles J. Brainerd (Ed.)*

Verbal Processes in Children:  
Progress in Cognitive Development Research  
*Charles J. Brainerd/Michael Pressley (Eds.)*

# **Children's Logical and Mathematical Cognition**

**Progress in Cognitive Development Research**

Edited by  
**Charles J. Brainerd**



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## Series Preface

For some time now, the study of cognitive development has been far and away the most active discipline within developmental psychology. Although there would be much disagreement as to the exact proportion of papers published in developmental journals that could be considered cognitive, 50% seems like a conservative estimate. Hence, a series of scholarly books to be devoted to work in cognitive development is especially appropriate at this time.

The *Springer Series in Cognitive Development* contains two basic types of books, namely, edited collections of original chapters by several authors, and original volumes written by one author or a small group of authors. The flagship for the Springer Series will be a serial publication of the “advances” type, carrying the subtitle *Progress in Cognitive Development Research*. Each volume in the *Progress* sequence will be strongly thematic, in that it will be limited to some well-defined domain of cognitive-developmental research (e.g., logical and mathematical development, semantic development). All *Progress* volumes will be edited collections. Editors of such collections, upon consultation with the Series Editor, may elect to have their books published either as contributions to the *Progress* sequence or as separate volumes. All books written by one author or a small group of authors will be published as separate volumes within the series.

A fairly broad definition of cognitive development is being used in the selection of books for this series. The classic topics of concept development, children’s thinking and reasoning, the development of learning, language development, and memory development will, of course, be included. So, however, will newer areas

such as social-cognitive development, educational applications, formal modeling, and philosophical implications of cognitive-developmental theory. Although it is anticipated that most books in the series will be empirical in orientation, theoretical and philosophical works are also welcome. With books of the latter sort, heterogeneity of theoretical perspective is encouraged, and no attempt will be made to foster some specific theoretical perspective at the expense of others (e.g., Piagetian versus behavioral or behavioral versus information processing).

C. J. Brainerd

## Preface

This is the first volume in a projected serial publication of the “advances” type that is to be concerned with the field of cognitive development, broadly defined. All volumes in this series will be published with the subtitle *Progress in Cognitive Development Research*. As this is the inaugural volume, some remarks about the guiding aims of the series are in order.

Like other “advances” publications, the principal goal is to present work that is on the growing tip of research in cognitive development and that meets the highest standards of our field. The format of individual volumes, however, will depart somewhat from the norm for “advances” series in psychology. A standard problem with most publications of this type is that only one or two contributions in any volume will be of interest to any given reader. The reason, of course, is that each volume, like individual issues of a technical journal, seeks to span the field as a whole. With the *Progress* series, however, no effort has been made to encompass cognitive-developmental research as a whole between the covers of single volumes. Instead, reasonable thematic structure has been imposed on each volume. For example, the theme of this book is children’s logical and mathematical cognition. The next three books in the series will deal with verbal processes in children, children’s learning, and new directions in cognitive-developmental theory, respectively. It is my hope that it will prove possible to identify themes that, on the one hand, will be sufficiently broad to be relevant to a significant proportion of the readership and that, on the other hand, will be sufficiently focused to interest any given reader in the contents of most of the contributions. To be sure, this will be a

delicate balancing act that, in all probability, will not always succeed. I strongly encourage investigators who have themes to suggest to contact me about guest editing a *Progress* volume.

Another difficulty that commonly arises with “advances” series is that of being committed to a one-volume-per-year publication schedule, usually with strict page limits. This means that during certain years meritorious work has to be excluded on grounds of space constraints. To avoid this, the *Progress* series will be flexible with respect to the number of volumes published per year. There will be two this year (the present volume and *Verbal Processes in Children*) and two next year (*Recent Advances in Cognitive-Developmental Theory* and *Learning in Children*). Thereafter, however, the number of volumes published in any one year will depend only on the number of provocative themes.

A second departure in format in this series concerns the style of chapters in individual volumes. In most books of this type, especially those in the classic areas of experimental psychology, the prototype contribution is a chapter written by a prominent investigator in which results and conclusions from a cumulative program of research are discussed. There are, I think, two objections to imposing this familiar structure on an “advances” series in cognitive development. First, although the scientific study of cognitive development is a broad and active field, it is relatively new in comparison with areas such as psychophysics, memory, learning, animal behavior, and perception. Consequently, the numbers of laboratories and investigators engaged in large-scale, cumulative research (as opposed to isolated studies) are as yet modest. I doubt that these numbers are sufficient to support an “advances” series along the aforementioned lines, though I could certainly be mistaken in this. Second and more important, this structure quite obviously tends to preclude contributions by new investigators. This would be particularly unfortunate for a series in cognitive development, because at the moment the field seems to be blessed with many promising young researchers from whom we have much to learn. It is my hope that the work of such newer contributors will figure prominently in the pages of this series. In the case of the present volume, for example, there is a roughly equal division between chapters written by younger researchers and those by more senior workers. In *Verbal Processes in Children*, the slant toward younger contributors is even more apparent.

In place of the traditional distinguished-researcher-discusses-recent-work chapters, *Progress* volumes will normally aim for a mix of three types of contributions: (a) reports of new findings, (b) reviews of extant literatures, and (c) theoretical essays. Contributions in category (a), which are represented by the chapters by Siegel and Saxe in this volume, are intended to cover comprehensive programs of research dealing with problems whose significance is widely acknowledged. Here, the sorts of articles that are typically found in the *Journal of Experimental Psychology: General* are roughly what I have in mind. Contributions in category (b), which are represented by the chapter by Fuson, Richards, and Briars and that by Hoemann and Ross in this volume, are intended to be interpretative reviews that deal with some well-defined literature. Thus, *Psychological Bulletin* reviews come closer to the intent of category (b) than do the general literature overviews that one



finds in, say, *Review of Child Development Research* or *Annual Review of Psychology*. Contributions in category (c), which are represented by the chapters by Acredolo and Brainerd in this volume, are intended to be serious attempts at integrative and explanatory theorizing, rather than speculative think pieces or critiques of the logical structure of some other theory. In other words, *Psychological Review* articles are nearer the mark than, say, *Behavioral and Brain Sciences* articles, or *Mind* articles, or *Human Development* articles.

Regarding the contents of this book, children's logical and mathematical cognition was selected as the theme for the first volume for reasons that are largely historical. It is this particular area of cognitive-developmental research that has been most closely connected to Piaget's theory, at least in the past. Since cognitive-developmental research owes its current prominence to the intensive interest in Piagetian theory that characterized the 1960s and early 1970s, it seems only fitting that the inaugural volume should deal with a topic on which Piaget's influence has been profound.

There are six chapters in all. Two of them focus on aspects of number development: Fuson, Richards, and Briars review the state of our knowledge of children's counting systems and report some new results from their laboratories. One of the most striking features of this chapter is that counting, which seems to be such a simple behavior to adults, turns out to have a remarkably complex structure. Saxe reports some recent findings from his ongoing studies of mathematical cognition among the Oksapmin tribes in Papua New Guinea. The Oksapmin have a unique numerical system based on body parts, and Saxe reports how this system is affected by contact with Western arithmetical ideas.

I should perhaps add that the fact that almost half the chapters are devoted to topics in number development is not accidental. At present, this seems to be the most active subdiscipline within the study of logical-mathematical development, and I would judge that at least half of the work that has appeared lately is somehow connected to number development. Since so much of the elementary school curriculum is devoted to transmitting basic numerical skills, the popularity of number-development research is hardly surprising.

The remaining chapters are by Acredolo, Hoemann and Ross, Siegel, and myself. In view of the enormous impact of conservation concepts on the study of logical-mathematical development, no volume devoted to this area can be considered complete without at least one chapter on conservation. Although conservation comes up in some of the other contributions, it is Acredolo's chapter that is principally concerned with these important concepts. Acredolo presents a new theory of the cognitive bases for conservation and summarizes the results of some previous experiments from which this theory evolved. In the chapter by Hoemann and Ross, the literature on children's probability concepts is reviewed. Since nearly all of the decisions that we make in everyday life are based on probabilistic information, probability concepts have long been extensively studied in the adult cognitive literature. Curiously, research on age changes in probability concepts has been much more sparse, with most of it concerned with testing one or another prediction from Piaget's theory. It is hoped that Hoemann and Ross' detailed,

integrative review of the literature will awaken interest in what is potentially a very instructive research domain.

Siegel considers the perennial question of how children's linguistic competence interacts with the assessment of their logical-mathematical competence. She describes nonverbal assessment techniques for a variety of competences that have been productive with normal children and in assessing the logical-mathematical competence of children with linguistic disabilities. My own chapter is concerned with another perennial question, namely, children's learning of logical and mathematical concepts. I outline a general learning system for certain concepts that is based on a rule-sampling interpretation of a family of Markovian processes, and some relevant experiments are reported. The ability of this framework to deal with such thorny issues as learning–development interactions, transfer effects, and age changes in information-processing capacity is also analyzed.

C. J. Brainerd

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