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FOREWORD

This IMA Volume in Mathematics and its Applications

LINEAR ALGEBRA FOR CONTROL THEORY

is based on the proceedings of a workshop which was an integral part of the 1991-92 IMA program on “Applied Linear Algebra”. We thank Paul Van Dooren and Bostwick Wyman for editing the proceedings. We also take this opportunity to thank the Army Research Office and the National Science Foundation, whose financial support made the workshop possible.

Avner Friedman

Willard Miller, Jr.

PREFACE

During the past decade, the interaction between control theory and linear algebra has been ever increasing, giving rise to new results in both areas. The purpose of this workshop was to develop further and nurture this cross-fertilization by bringing together control theorists and linear algebraists for an exchange of problems, ideas, and techniques. The cross-fertilization between control and linear algebra can be found in the following subfields:

1. Numerical Linear Algebra for Control. One of the most profound impacts of linear algebra in control theory has been in the computational and implementational aspects, where numerical linear algebraic algorithms have strongly influenced the ways in which many problems are being solved. The interaction between the numerical analysis and control community has led to important new results in the solution of special eigenvalue and singular value problems, in calculating invariants and canonical forms, in solving matrix Riccati, Lyapunov, and Sylvester equations and the like. It is still a very active area as new problems keep emerging (see also 3,4, and 5).

2. Canonical Forms and Invariants. Since the introduction of state space models in control theory, canonical forms and invariants have been very important. While there is a clear connection with classical canonical forms such as the Jordan, Kronecker and Smith forms, the problems here are yet more intricate. This is due to the presence of four mappings (often called A,B,C,D) and to a large class of group actions that can be defined on them. Recent interests here include, e.g., the group of orthogonal transforms. Also, for transfer matrix models, new invariants have been defined and relations with other models are being looked at.

3. Ring-theoretic Methods in Linear Control. Many algebraic issues in linear algebra are properly treated through the use of ring and module-theoretic techniques. In the context of control theory, such techniques have already proved extremely useful in the study of various system design questions such as those dealing with invertibility (abstract pole and zero modules), in the study of certain perturbation models (systems with small parameters modeled through matrices over parameter rings), and in questions such as the study of pole shifting for delay and other infinite-dimensional systems through systems over operator rings.

4. Matrix Theory in Control. The models used in control theory are in fact all well-studied objects in matrix theory: rational matrices, polynomial matrices, and pencils. Certain expansions lead to Hankel and Toeplitz matrices, possibly block versions of those. This connection attracted the attention of several researchers in matrix and operator theory,

and it resulted in a wealth of new results in matrix theory, which are also of importance to control people. Typical examples are factorizations of rational matrices, embedding and completion problems, invariants, and also canonical forms (see also 2).

5. H^∞ control. Robust control has been one of the most active areas of control during the last five years. The mathematical problems encountered there bear relation to several other fields, but many of the most recent results lie, in fact, in linear algebra. The current trend is to formulate these problems in terms such of state space models, which then leads to special matrix equations. Typical examples are coupled matrix Riccati equations and special singular value problems. Other issues of H^∞ control are also related to the theory of matrix valued functions (see 4), and computational aspects also play an important role here (see 1).

Around each of these subject areas a 1 day session was organized during the workshop. Special thanks ought to go to Angelika Bunse-Gerstner, Volker Mehrmann, Diederich Hinrichsen, Eduardo Sontag, Bostwick Wyman, Leiba Rodman, and, finally, Andre Ran, who skillfully put together nice sessions on each of these topics.

The workshop was a peculiar one in the sense that the five themes looked almost disjoint. The challenge of the workshop was to present the latest results in these areas and to find points of common interest. The results were very positive indeed. Throughout the meeting one could see how the same problems occurred in different sessions and were being tackled from a different point of view. This interaction was also felt strongly in the corridors of the 5th floor of the IMA, where IMA visitors defended their way of looking at a particular common point of interest.

Altogether the main strength of the meeting was its multidisciplinary nature and the cross fertilization resulting from this. The relaxed atmosphere offered by the IMA also played a key role in making this happen. For making this possible, we wish to thank Willard Miller and Avner Friedman from the IMA, and also Dick Brualdi, who was a key figure in the organization of the Applied Linear Algebra Year held at the IMA.

Paul Van Dooren
Bostwick Wyman

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