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17

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Preface

This book has grown out of lectures and courses in calculus of variations and optimization taught for many years at the University of Michigan to graduate students at various stages of their careers, and always to a mixed audience of students in mathematics and engineering. It attempts to present a balanced view of the subject, giving some emphasis to its connections with the classical theory and to a number of those problems of economics and engineering which have motivated so many of the present developments, as well as presenting aspects of the current theory, particularly value theory and existence theorems. However, the presentation of the theory is connected to and accompanied by many concrete problems of optimization, classical and modern, some more technical and some less so, some discussed in detail and some only sketched or proposed as exercises.

No single part of the subject (such as the existence theorems, or the more traditional approach based on necessary conditions and on sufficient conditions, or the more recent one based on value function theory) can give a sufficient representation of the whole subject. This holds particularly for the existence theorems, some of which have been conceived to apply to certain large classes of problems of optimization.

For all these reasons it is essential to present many examples (Chapters 3 and 6) before the existence theorems (Chapters 9 and 11–16), and to investigate these examples by means of the usual necessary conditions, sufficient conditions, and value function theory.

This book only considers nonparametric problems of the calculus of variations in one independent variable and problems of optimal control monitored by ordinary differential equations. Multidimensional problems monitored by partial differential equations, parametric problems with simple and multiple integrals, parametric problems of optimal control, and related questions of nonlinear integration will be presented elsewhere.

Chapter 1 is introductory. The many types of problems of optimization are reviewed and their intricate relationships illustrated.

Chapter 2 presents the necessary conditions, the sufficient conditions, and the value function theory for classical problems of the calculus of variations. In particular, the Weierstrass necessary condition is being studied as a necessary condition for lower semicontinuity on a given trajectory.

Chapter 3 consists mainly of examples. In particular, it includes points of Ramsey's theory of economic growth, and points of theoretical mechanics.

Chapters 4 and 5 deal with problems of optimal control. They contain a statement of the necessary condition, a detailed discussion of the transversality relation in its generality, a discussion of Bellman's value function theory, and a statement of Boltyanskii's sufficient condition in terms of regular synthesis.

Chapter 6 consists mainly of examples. In particular, points of the neoclassical theory of economic growth are also studied.

Chapter 7 presents two proofs of the necessary condition for problems of optimal control.

Chapter 8 contains preparatory material for existence theorems, in particular, Kuratowski's and Ryll-Nardzewski's selection theorems, McShane's and Warfield's implicit function theorem, and some simple forms of the lower closure theorem for uniform convergence.

Chapter 9 deals with existence theorems for problems of optimal control with continuous data and compact control space. These are essentially Filippov's existence theorems. The proofs in this chapter are designed to be elementary in the sense that mere uniform convergence is involved, whereas in Chapters 10 and 11 use is made of weak convergence in L_1.

Chapter 10 presents the Banach–Saks–Mazur theorem, the Dunford-Pettis theorem, and closure, lower closure, and lower semicontinuity theorems for weak convergence in L_1.

Chapter 11 deals with existence theorems based on weak convergence. Existence theorems are proved for Lagrange problems with an integrand which is an extended function, and then existence theorems are derived for problems of optimal control. Moreover, existence theorems are proved for problems with comparison functionals, for isoperimetric problems, and specifically for problems which are linear in the derivatives, or in the controls. In particular, this chapter contains a present day version of the theorem established by Tonelli in 1914 for problems with a uniform growth property.

In Chapter 12 existence theorems are presented where a growth assumption fails at the points of a "slender" set. In Chapter 13 existence theorems under numerous analytical conditions are studied. Chapter 14 deals with existence theorems for problems without growth assumptions. Chapter 15 presents theorems based on mere pointwise convergence. Chapter 16 deals with Neustadt-type existence theorems for problems with no convexity assumptions.

Chapter 17 covers a few points of convex analysis including duality, and the equivalence of a certain concept of upper semicontinuity for sets with the concept of seminormality of Tonelli and McShane for functions, and suitable properties in terms of convex analysis.

Chapter 18 covers questions of approximation of usual and generalized trajectories.

Each chapter contains examples and exercises. Bibliographical notes at the end of each chapter provide some historical background and direct the reader to the literature in the field.

A number of parts in this book are in smaller print so as to facilitate, at a first reading, a faster perusal. The small-print passages include most of the examples and remarks, several of the complementary considerations, and a number of the more technical proofs.

I wish to thank the many associates and graduate students who, with their remarks and suggestions upon reading these notes, have contributed so much to make this presentation a reality.

Finally, I wish to express my appreciation to Springer-Verlag for their accomplished handling of the manuscript, their understanding and patience.

Contents

To Isotta, always