REPRESENTATIONS OF DISCRETE FUNCTIONS

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EDITED BY

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PREFACE

In August 27-29, 1995, the IFIP WG 10.5 Workshop on Applications of the Reed-Muller Expansions in Circuit Design (Reed-Muller '95) was held in Maku-hari, Chiba, Japan. Many papers focused on the representation of discrete functions. They were so instructive that we collected the most outstanding papers in a book form.

This book consists of 13 chapters. To enhance their readability, all chapters here were reviewed again and were revised. In the original papers in the proceedings, authors used their own terminology. Thus, the same notion was represented by different terminology. In the book version, however, we have insisted on a unified terminology. We include three chapters showing standard terminology. Additional examples and illustrations designed to increase the readers' understandings were also incorporated in the book version. To make the book more comprehensive, we also included two relevant chapters, which were presented at the International Workshops on Logic Synthesis. Exercises prepared by editors will make this book appropriate for a textbook of a course.

This book deals with the following topics: Binary decision diagrams (BDDs), multi-terminal binary decision diagrams (MTBDDs), edge-valued binary decision diagrams (EVBDDs), functional decision diagrams (FDDs), Kronecker decision diagrams (KDDs), binary moment diagrams (BMDs), spectral transform decision diagrams (STDDs), ternary decision diagrams (TDDs), spectral transformation of logic functions, other transformations of logic functions, EXOR-based two-level expressions, FPRM minimization with TDDs and MTBDDs, complexity theories on FDDs, multi-level logic synthesis, and complexity of three-level logic networks.

In this book, a **discrete function** denotes a mapping $S \rightarrow L$, where S and L are finite non-empty sets. A **switching function** denotes a discrete function, where $S = \{0, 1\}^n$ and $L = \{0, 1\}$.

Figure 1 Relation of chapters.

Overview of the Book

This book consists of 13 chapters. Fig. 1 shows the relation of these chapters. The first three chapters are introductions. They contain many illustrations to clarify concepts presented in the text. It is recommended to read these three chapter first. After this, the other chapters will be easy to read.

Chapter 1 introduces BDDs, and their variants: it surveys the techniques for BDDs, and shows the relations between different types of decision diagrams.

Chapter 2 introduces EXOR-based representations: it compares complexities of various representations.

Chapter 3 shows relation of various decision diagrams: One decision diagram represents a function f , and at the same time, the spectrum S_f . For example, a BDD represents f , and at the same time, the Reed-Muller spectrum of f . It also shows methods to represent arithmetic functions by using decision diagrams. This chapter is related to Chapters 4, 5, and 6.

Chapter 4 shows applications of MTBDDs. It shows an MTBDD-based computation of spectral transformations, which is a revolutionary idea. With this technique, we can compute the spectra of a logic function with more than 200 variables. This chapter also proposes HDDs (hybrid decision diagrams).

Chapter 5 introduces EVBDDs, which are useful to represent both Boolean and arithmetic operations. This chapter shows EVBDD algorithms for binary operations. An extension of EVBDDs is also described.

Chapter 6 presents arithmetic transformations, which are useful for verification of large networks.

Chapter 7 considers OKFDDs (KDDs). OKFDDs are generalization of BDDs and FDDs, and can represent logic functions more compactly than BDDs and FDDs.

Chapter 8 shows a method for exact minimization of FPRMs. With this method, we could obtain an exact minimum FPRM with 94 inputs. MTBDDs, and MTTDDs are the key data structure in this method.

Chapter 9 shows a multi-level logic synthesis method. It considers both AND-OR and EXOR-based designs.

The next two chapters deal with complexity issues of FDDs.

Chapter 10 considers the satisfiability problems. It shows that the counting the satisfying assignments in an OFDD cannot be solved in polynomial time unless $NP = P$.

Chapter 11 considers the complexity of changing the order of input variables. It also shows that the replacement of variables with constants can cause an exponential blow up of the FDD size.

Chapter 12 presents ternary decision diagrams (TDDs). A TDD represents a logic expression rather than a logical function. A BDD is a special case of a TDD, where only two edges representing f_0 and f_1 are used. There is a $2n$ variable function whose size of the TDD is $2n^2 + 2$, while the size of the BDD is 2^{n+1} . TDD can be a mathematical model for analyzing the complexity of Boolean functions.

Chapter 13 deals the complexity of OR-AND-OR three-level logic networks. Any logic functions with $n = 2r$ variables are represented with at most 2^{r+1} gates.

Unlike the other chapters, preliminary versions of last two chapters were presented at the International Workshop on Logic Synthesis in May 1993, and 1989, respectively.

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Students of Sasao's Laboratory made important suggestions to improve the readability of the chapters during seminars using the preliminary version of the book.

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