

Lectures on Boolean Algebras

Lectures on BOOLEAN ALGEBRAS

Paul R. Halmos

Springer-Verlag New York • Heidelberg • Berlin
1974

PAUL R. HALMOS
Indiana University
Department of Mathematics
Swain Hall East
Bloomington, Indiana 47401

AMS Subject Classifications (1970)
02J05, 06A40

Library of Congress Cataloging in Publication Data

Halmos, Paul Richard, 1914-
Lectures on Boolean algebras.

Reprint of the ed. published by Van Nostrand, New York, which was issued as no. 1 of Van Nostrand mathematical studies.

1. Algebra, Boolean. I. Title.

[QA10.3.H34 1974] 511.32 74-10632

ISBN-13:978-0-387-90094-0

All rights reserved.

No part of this book may be translated or reproduced in any form without written permission from Springer-Verlag.

© 1963 by D. Van Nostrand Company, Inc. and 1974 by Springer-Verlag New York Inc.

ISBN-13:978-0-387-90094-0 e-ISBN-13:978-1-4612-9855-7
DOI: 10.1007/978-1-4612-9855-7

Preface

IN 1959 I lectured on Boolean algebras at the University of Chicago. A mimeographed version of the notes on which the lectures were based circulated for about two years; this volume contains those notes, corrected and revised. Most of the corrections were suggested by Peter Crawley. To judge by his detailed and precise suggestions, he must have read every word, checked every reference, and weighed every argument, and I am very grateful to him for his help. This is not to say that he is to be held responsible for the imperfections that remain, and, in particular, I alone am responsible for all expressions of personal opinion and irreverent viewpoint.

P. R. H.

Ann Arbor, Michigan

January, 1963

Contents

<i>Section</i>		<i>Page</i>
1	Boolean rings	1
2	Boolean algebras	3
3	Fields of sets	9
4	Regular open sets	12
5	Elementary relations	17
6	Order	21
7	Infinite operations	25
8	Subalgebras	31
9	Homomorphisms	35
10	Free algebras	40
11	Ideals and filters	47
12	The homomorphism theorem	52
13	Boolean σ -algebras	55
14	The countable chain condition	61
15	Measure algebras	64
16	Atoms	69
17	Boolean spaces	72
18	The representation theorem	77
19	Duality for ideals	81
20	Duality for homomorphisms	84
21	Completion	90
22	Boolean σ -spaces	97
23	The representation of σ -algebras	100
24	Boolean measure spaces	104
25	Incomplete algebras	109
26	Products of algebras	115
27	Sums of algebras	119
28	Isomorphisms of factors	122
29	Isomorphisms of countable factors	126
30	Retracts	130
31	Projective algebras	137
32	Injective algebras	140
	Epilogue	144
	Index	145