Controlled Stochastic Processes

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Preface

The theory of controlled processes is one of the most recent mathematical theories to show very important applications in modern engineering, particularly for constructing automatic control systems, as well as for problems of economic control. However, actual systems subject to control do not admit a strictly deterministic analysis in view of random factors of various kinds which influence their behavior. Such factors include, for example, random noise occurring in the electrical system, variations in the supply and demand of commodities, fluctuations in the labor force in economics, and random failures of components on an automated line. The theory of controlled processes takes the random nature of the behavior of a system into account. In such cases it is natural, when choosing a control strategy, to proceed from the average expected result, taking note of all the possible variants of the behavior of a controlled system.

An extensive literature is devoted to various economic and engineering systems of control (some of these works are listed in the Bibliography). However, as of now there is no text which adequately covers the general mathematical theory of controlled processes. The authors of this monograph have attempted to fill this gap.

In this volume the general theory of discrete-parameter (time) controlled processes (Chapter 1) and those with continuous-time (Chapter 2), as well as the theory of controlled stochastic differential equations (Chapter 3), are presented. In this book, the notion of a controlled stochastic object serves as a departing basic concept; this allows us to substantially avoid the difficulties associated with continuous-parameter processes that are so familiar to specialists in the field. The traditional problems of optimal stopping rules for processes and the derivation of Bellman's equations and their application to the construction of optimal equations for controlled Markov

processes are also examined in this volume. The authors have attempted to minimize the use of advanced mathematical methods; however, a knowledge of the basic notions of the theory of stochastic processes and the rudiments of measure theory and functional analysis are presupposed.

The authors would appreciate receiving comments and suggestions from the readers.

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