Learning Algorithms Theory and Applications

S. Lakshmivarahan

Learning Algorithms

Theory and Applications

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Prof. S. Lakshmivarahan
University of Oklahoma
School of Electrical Engineering
and Computer Science
Norman, Oklahoma 73019

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DEDICATED TO
MY
FATHER and MOTHER

Who taught me many useful algorithms.

PREFACE

Learning constitutes one of the most important phase of the whole psychological processes and it is essential in many ways for the occurrence of necessary changes in the behavior of adjusting organisms. In a broad sense influence of prior behavior and its consequence upon subsequent behavior is usually accepted as a definition of learning. Till recently learning was regarded as the prerogative of living beings. But in the past few decades there have been attempts to construct learning machines or systems with considerable success.

This book deals with a powerful class of learning algorithms that have been developed over the past two decades in the context of learning systems modelled by finite state probabilistic automaton. These algorithms are very simple iterative schemes. Mathematically these algorithms define two distinct classes of Markov processes with unit simplex (of suitable dimension) as its state space. The basic problem of learning is viewed as one of finding conditions on the algorithm such that the associated Markov process has prespecified asymptotic behavior. As a prerequisite a first course in analysis and stochastic processes would be an adequate preparation to pursue the development in various chapters.

Automaton models and algorithms for learning, which is the central theme of the present book, have been widely accepted as one of the fundamental approaches to machine intelligence. In spite of this wide spread acceptability most of the published material on this topic to date are confined to articles in research journals or isolated chapters in anthologies and as such there is no one single text book or monograph on this topic. This book is intended to fill this gap.

The book consists of two parts: Part I develops the theory of design of learning algorithms. In particular it deals with the convergence behavior of various classes of learning algorithms. Part II describes the application of these algorithms to decentralized decision making problems with incomplete information. More specifically, we deal with the applications to two person zero sum sequential games with incomplete information, two person team problem with incomplete information and Markov decision problem with incomplete information.

Our aim is to provide a unified framework and a comprehensive treatment of most of the results pertaining to both the theory and applications known to date. Interestingly enough, much of the results presented in Part II came into existence only during the past three to four years.

Basically, the book arose out of the notes written by the author for a series of lectures delivered at the Institute of Applied Mathematics, University of Bonn, W. Germany during January 1980 under the auspices of their Sonderforschungsbereiches. A major part of an earlier version was also used for the graduate course CS 5070, Seminar in Problem Solving at the School of Electrical Engineering and Computer Science, University of Oklahoma during Spring Semester 1980.

The author wishes to record his sincere appreciation and thanks to Professors R. Leis, W. Vogel, V. Herkenrath and D. Kalin of the University of Bonn for arranging this visit and their hospitality. Special thanks are due to Professors Herkenrath and Kalin with whom the author has spent countless hours discussing various aspects of both the theory and applications of learning algorithms. Dr. Herkenrath read through the entire manuscript and has suggested various improvements. It was Professor Radu Theodorescu of Laval University who introduced the author to the University of Bonn. Our thanks are due to him and his collaborator

Professor M. Iosifescu for their constant encouragement. Professor M.A.L. Thathachar initiated the author into the field of learning algorithms when he was a graduate student at the Department of Electrical Engineering, Indian Institute of Science, Bangalore, India. Ever since that time, he has been a great source of inspiration and continued guidance. His influence will be felt throughout the book and the author is deeply indebted to him. Much of the work in Part II (Chapters 5 through 7) was carried out when the author was at Yale University during 1976 - 1978 under the direction of Professor K.S. Narendra. It was Professor Narendra who introduced the author into the exciting area of learning approach to decentralized decision making with incomplete information. To him we are very grateful for his continued advice and support. In addition, quite a variety of people have helped the author in many different ways. Professor H.J. Kushner helped to clarify many intricacies of different classes of learning algorithms. In fact, his well known counter example marked a definite turning point (for the better) in our research work. Professors K.S. Fu, M.D. Srinath and H. Aso have read through the various portions of the manuscript and have suggested various modifications. Professors M.E. Council and S.K. Kahng (past and present directors) of the School of Electrical Engineering and Computer Science, University of Oklahoma have extended full support for the completion of this work. The Mathematics editorial staff of Springer Verlag, Inc. New York handled this project with great care and patience. Our sincere appreciation and thanks are due to all these above individuals. Thanks are also due to Betty Sudduth, Vici Chance and Ronda Sharp for typing various portions of the manuscript.

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Norman, Oklahoma

July 1981

S. Lakshmivarahan

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