

Applied Mathematical Sciences

EDITORS

Fritz John
*Courant Institute of
Mathematical Sciences*
New York University
New York, N.Y. 10012

Lawrence Sirovich
*Division of
Applied Mathematics*
Brown University
Providence, R.I. 02912

Joseph P. LaSalle
*Division of
Applied Mathematics*
Lefschetz Center
for Dynamical Systems
Providence, R.I. 02912

ADVISORS

H. Cabannes University Paris-VI

J. Marsden University of California at Berkeley

J.K. Hale Brown University

G.B. Whitman California Institute of Technology

J. Keller Stanford University

EDITORIAL STATEMENT

The mathematization of all sciences, the fading of traditional scientific boundaries, the impact of computer technology, the growing importance of mathematical-computer modelling and the necessity of scientific planning all create the need both in education and research for books that are introductory to and abreast of these developments.

The purpose of this series is to provide such books, suitable for the user of mathematics, the mathematician interested in applications, and the student scientist. In particular, this series will provide an outlet for material less formally presented and more anticipatory of needs than finished texts or monographs, yet of immediate interest because of the novelty of its treatment of an application or of mathematics being applied or lying close to applications.

The aim of the series is, through rapid publication in an attractive but inexpensive format, to make material of current interest widely accessible. This implies the absence of excessive generality and abstraction, and unrealistic idealization, but with quality of exposition as a goal.

Many of the books will originate out of and will stimulate the development of new undergraduate and graduate courses in the applications of mathematics. Some of the books will present introductions to new areas of research, new applications and act as signposts for new directions in the mathematical sciences. This series will often serve as an intermediate stage of the publication of material which, through exposure here, will be further developed and refined. These will appear in conventional format and in hard cover.

MANUSCRIPTS

The Editors welcome all inquiries regarding the submission of manuscripts for the series. Final preparation of all manuscripts will take place in the editorial offices of the series in the Division of Applied Mathematics, Brown University, Providence, Rhode Island.

SPRINGER-VERLAG NEW YORK INC., 175 Fifth Avenue, New York, N. Y. 10010

Printed in U.S.A.

Applied Mathematical Sciences | Volume 32

Theodor Meis
Ulrich Marcowitz

Numerical Solution of Partial Differential Equations



Springer-Verlag
New York Heidelberg Berlin

Theodor Meis
Mathematisches Institut
der Universität zu Köln
Weyertal 86-90
5000 Köln 41
Federal Republic of Germany

Ulrich Marcowitz
Mathematisches Institut
der Universität zu Köln
Weyertal 86-90
5000 Köln 41
Federal Republic of Germany

Translated by Peter R. Wadsack, University of Wisconsin.

AMS Subject Classifications: 65MXX, 65NXX, 65P05

Library of Congress Cataloging in Publication Data

Meis, Theodor.

Numerical solution of partial differential equations.

(Applied mathematical sciences; 32)

Translation of *Numerische Behandlung partieller
Differentialgleichungen*.

Bibliography: p.

Includes index.

1. Differential equations, Partial—Numerical
solutions. I. Marcowitz, Ulrich, joint author.

II. Title. III. Series.

QA1.A647 vol. 32 [QA374] 510s [515.3'53]
80-26520

English translation of the original German edition *Numerische Be-
handlung Partieller Differentialgleichungen* published by Springer-
Verlag Heidelberg © 1978.

All rights reserved.

No part of this book may be translated or reproduced in any form
without written permission from Springer-Verlag.

© 1981 by Springer-Verlag New York Inc.

9 8 7 6 5 4 3 2 1

ISBN-13: 978-0-387-90550-1

e-ISBN-13: 978-1-4612-5885-8

DOI: 10.1007/978-1-4612-5885-8

PREFACE

This book is the result of two courses of lectures given at the University of Cologne in Germany in 1974/75. The majority of the students were not familiar with partial differential equations and functional analysis. This explains why Sections 1, 2, 4 and 12 contain some basic material and results from these areas.

The three parts of the book are largely independent of each other and can be read separately. Their topics are: initial value problems, boundary value problems, solutions of systems of equations. There is much emphasis on theoretical considerations and they are discussed as thoroughly as the algorithms which are presented in full detail and together with the programs. We believe that theoretical and practical applications are equally important for a genuine understanding of numerical mathematics.

When writing this book, we had considerable help and many discussions with H. W. Branca, R. Esser, W. Hackbusch and H. Multhei. H. Lehmann, B. Müller, H. J. Niemeyer, U. Schulte and B. Thomas helped with the completion of the programs and with several numerical calculations.

Springer-Verlag showed a lot of patience and understanding during the course of the production of the book. We would like to use the occasion of this preface to express our thanks to all those who assisted in our sometimes arduous task.

Cologne, Fall 1980
Th. Meis
U. Marcowitz

CONTENTS

	Page
PART I. INITIAL VALUE PROBLEMS FOR HYPERBOLIC AND PARABOLIC DIFFERENTIAL EQUATIONS.	1
1. Properly posed initial value problems	1
2. Types and characteristics	19
3. Characteristic methods for first order hyperbolic systems	31
4. Banach spaces	40
5. Stability of difference methods	55
6. Examples of stable difference methods	73
7. Inhomogeneous initial value problems.	89
8. Difference methods with positivity properties . .	97
9. Fourier transforms of difference methods.	119
10. Initial value problems in several space variables	168
11. Extrapolation methods	192
PART II. BOUNDARY VALUE PROBLEMS FOR ELLIPTIC DIFFERENTIAL EQUATIONS	207
12. Properly posed boundary value problems.	207
13. Difference methods.	229
14. Variational methods	270
15. Hermite interpolation and its application to the Ritz method	290
16. Collocation methods and boundary integral methods	317
PART III. SOLVING SYSTEMS OF EQUATIONS.	334
17. Iterative methods for solving systems of linear and nonlinear equations	334
18. Overrelaxation methods for systems of linear equations	363
19. Overrelaxation methods for systems of nonlinear equations	383
20. Band width reduction for sparse matrices.	402
21. Buneman Algorithm	417
22. The Schröder-Trottenberg reduction method	426
APPENDICES: FORTRAN PROGRAMS.	444
Appendix 0: Introduction.	444
Appendix 1: Method of Massau.	447
Appendix 2: Total implicit difference method for solving a nonlinear parabolic differential equation	459
Appendix 3: Lax-Wendroff-Richtmyer method for the case of two space variables	469
Appendix 4: Difference methods with SOR for solving the Poisson equation on nonrectangular regions	484
Appendix 5: Programs for band matrices.	503
Appendix 6: The Buneman algorithm for solving the Poisson equation.	522

	Page
BIBLIOGRAPHY	532
INDEX.	538