



Progress in Nonlinear Differential Equations and Their Applications

Volume 18

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The Heat Kernel
Lefschetz Fixed Point
Formula for the
Spin-c Dirac Operator

Birkhäuser
Boston • Basel • Berlin

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Mathematisch Instituut
Universiteit Utrecht
3508 TA Utrecht
The Netherlands

Library of Congress Cataloging-in-Publication Data

Duistermaat, J. J. (Johannes Jisse), 1942-

The heat kernel Lefschetz fixed point formula for the spin-c dirac operator / J. J. Duistermaat

p. cm. -- (Progress in nonlinear differential equations and their applications ; v. 18)

Includes bibliographical references and index.

1. Almost complex manifolds. 2. Operator theory. 3. Dirac equation. 4. Differential topology. 5. Mathematical physics.

I. Title. II. Series.

QC20.7.M24D85 1995

95-25828

515'.7242--dc20

CIP

Printed on acid-free paper

Birkhäuser 

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ISBN-13: 978-1-4612-5346-4

e-ISBN-13: 978-1-4612-5344-0

DOI: 10.1007/978-1-4612-5344-0

Typeset from author's disk by TeXniques, Boston, MA

Printed and bound by Quinn-Woodbine, Woodbine, NJ

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Preface

When visiting M.I.T. for two weeks in October 1994, Victor Guillemin made me enthusiastic about a problem in symplectic geometry which involved the use of the so-called spin-c Dirac operator. Back in Berkeley, where I had spent a sabbatical semester¹, I tried to understand the basic facts about this operator: its definition, the main theorems about it, and their proofs. This book is an outgrowth of the notes in which I worked this out. For me this was a great learning experience because of the many beautiful mathematical structures which are involved.

I thank the Editorial Board of Birkhäuser, especially Haim Brézis, for suggesting the publication of these notes as a book. I am also very grateful for the suggestions by the referees, which have led to substantial improvements in the presentation. Finally I would like to express special thanks to Ann Kostant for her help and her prodding me, in her charming way, into the right direction.

J.J. Duistermaat

Utrecht, October 16, 1995.

¹Partially supported by AFOSR Contract AFO F 49629-92