

# Lecture Notes in Statistics

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# Lecture Notes in Statistics

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J. Hartigan, and K. Krickeberg

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Søren Johansen

Functional Relations,  
Random Coefficients,  
and Nonlinear Regression  
with Application  
to Kinetic Data

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## PREFACE

These notes on regression give an introduction to some of the techniques that I have found useful when working with various data sets in collaboration with Dr. S. Keiding (Copenhagen) and Dr. J.W.L. Robinson (Lausanne).

The notes are based on some lectures given at the Institute of Mathematical Statistics, University of Copenhagen, 1978-81, for graduate students, and assumes a familiarity with statistical theory corresponding to the book by C.R. Rao: "Linear Statistical Inference and its Applications". Wiley, New York (1973).

The mathematical tools needed for the algebraic treatment of the models are some knowledge of finite dimensional vector spaces with an inner product and the notion of orthogonal projection. For the analytic treatment I need characteristic functions and weak convergence as the main tools.

The most important statistical concepts are the general linear model for Gaussian variables and the general methods of maximum likelihood estimation as well as the likelihood ratio test.

All these topics are presented in the above mentioned book by Rao and the reader is referred to that for details.

For convenience a short appendix is added where the fundamental concepts from linear algebra are discussed.

It is a pleasure to thank the Danish Society for Theoretical Statistics for inviting me to present a survey of the contents at the 9<sup>th</sup> Nordic Conference on Mathematical Statistics at Aalborg University Center, June 7-10, 1982, thereby providing the formal necessity for finishing the notes.

I also owe thanks to Vivi Nielsen for efficient typing and critical reading of the manuscript, and to Birthe Lykke Thomsen, Poul Thyregod and Jørgen Hilden for useful comments to one of the first versions.

Copenhagen, November 1933

Søren Johansen

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