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Modern Concepts and Theorems of Mathematical Statistics

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This book is dedicated to Tanya and Jacqueline

Preface

With the rapid progress and development of mathematical statistical methods, it is becoming more and more important for the student, the instructor, and the researcher in this field to have at their disposal a quick, comprehensive, and compact reference source on a very wide range of the field of modern mathematical statistics. This book is an attempt to fulfill this need and is encyclopedic in nature. It is a useful reference for almost every learner involved with mathematical statistics at any level, and may supplement any textbook on the subject. As the primary audience of this book, we have in mind the beginning busy graduate student who finds it difficult to master basic modern concepts by an examination of a limited number of existing textbooks. To make the book more accessible to a wide range of readers I have kept the mathematical language at a level suitable for those who have had only an introductory undergraduate course on probability and statistics, and basic courses in calculus and linear algebra. No sacrifice, however, is made to dispense with rigor. In stating theorems I have not always done so under the weakest possible conditions. This allows the reader to readily verify if such conditions are indeed satisfied in most applications given in modern graduate courses without being lost in extra unnecessary mathematical intricacies. The book is not a mere dictionary of mathematical statistical terms. It is also expository in nature, providing examples and putting emphasis on theorems, limit theorems, comparison of different statistical procedures, and statistical distributions. The various topics are covered in appropriate details to give the reader enough confidence in himself (herself) which will then allow him (her) to consult the references given in the Bibliography for proofs, more details, and more applications. At the end of various sections of the book references are given where proofs and/or further details may be found. No attempt is made here to supply historical details on who did what, when. Accordingly, I apologize to any colleague whose name is not found in the list of references or whose name may not appear attached to a theorem or to a statistical procedure. All that should matter to the reader is to obtain quick and precise information on the technical aspects he or she is seeking. To benefit as much as possible from the book, it is advised to consult first the Contents on a given topic, then the Subject Index, and then the section on Notations. Both the Contents and the Subject Index are quite elaborate.

We hope this book will fill a gap, which we feel does exist, and will provide a useful reference to all those concerned with mathematical statistics.

E. B. M.

Contents

Some Notations

$A \subseteq B$: A subset of B and may include equality.

$A \nsubseteq B$: A subset of B and excludes equality.

A^c: complement of A.

$$\binom{n}{x} \equiv \frac{n!}{(n-x)!\,x!}, \qquad n! \equiv \prod_{i=1}^{n} i, \qquad 0! \equiv 1.$$

$I_n = [\delta_{ij}]_{n \times n}$, $(n \times n)$ matrix, and where δ_{ij} is the Kronecker delta, i.e., $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ii} = 1$, $i, j = 1, \ldots, n$.

$c_n = O(n^\delta)$, i.e., $\lim_{n\to\infty} |n^{-\delta} c_n| < \infty$. Sometimes we also write $\lim_{n\to\infty} c_n = O(n^\delta)$.

Gamma function: $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\, dx$, Re $z > 0$.

$$\Gamma(n) = (n-1)!,\ n = 1, 2, \ldots.$$

$\Phi(t)$: Characteristic function.

Standard normal distribution: $\phi(z) = \int_{-\infty}^{z} e^{-x^2/2}\, dx/\sqrt{2\pi}$.

If Z is a random variable having a standard normal distribution, we use the notation: $P[Z \geqslant Z_\alpha] = \alpha$.

If χ^2 is a random variable having a chi-square distribution of v degrees of freedom, we use the notation: $P[\chi^2 \geqslant \chi_\alpha^2(v)] = \alpha$.

If T is a random variable having a Student distribution of v degrees of freedom, we use the notation: $P[T \geqslant T_\alpha(v)] = \alpha$.

If F is a random variable having a Fisher's F-distribution of v_1 and v_2 degrees of freedom, we use the notation: $P[F \geqslant F_\alpha(v_1, v_2)] = \alpha$.

$E[X] = \mu_X$ or just μ (mean); $\sigma^2(X) = E[(X - \mu)^2]$ or just σ^2 (variance); $E[X - \mu)^3]/\sigma^3 = \gamma_1$ (coefficient of skewness); $E[(X - \mu)^4]/\sigma^4 - 3 = \gamma_2$ (kurtosis) adjusted so that $\gamma_2 = 0$ for the normal distribution.

"X has a $N(\mu, \sigma^2)$ distribution" read "X has a normal distribution with mean μ and variance σ^2." "$\mathbf{X}$ has a $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution" read "$\mathbf{X}$ has a multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$."