

# Stability of Time Dependent and Spatially Varying Flows

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# Introduction

This volume is the collection of papers presented at the workshop on “The Stability of Spatially Varying and Time Dependent Flows” sponsored by the Institute for Computer Applications in Science and Engineering (ICASE) and NASA Langley Research Center (LaRC) during August 19–23, 1985. The purpose of this workshop was to bring together some of the experts in the field for an exchange of ideas to update the current status of knowledge and to help identify trends for future research. Among the invited speakers were D.M. Bushnell, M. Goldstein, P. Hall, Th. Herbert, R.E. Kelly, L. Mack, A.H. Nayfeh, F.T. Smith, and C. von Kerczek. The contributed papers were by A. Bayliss, R. Bodonyi, S. Cowley, C. Grosch, S. Lekoudis, P. Monkewitz, A. Patera, and C. Streett.

In the first article, Bushnell provides a historical background on laminar flow control (LFC) research and summarizes the crucial role played by stability theory in LFC system design. He also identifies problem areas in stability theory requiring further research from the view-point of applications to LFC design. It is an excellent article for theoreticians looking for some down-to-earth applications of stability theory.

The next seven articles deal with the stability of spatially varying incompressible flows, followed by two articles on compressible flows. Nayfeh, who is among the first to study the stability of nonparallel flows, reviews rather comprehensively the theoretical studies of the secondary instability appearing in the early stages of transition in a boundary layer flow. He divides these investigations into two types—the mutual interaction type including resonant and nonresonant interactions, and the parametric type. He compares and contrasts these approaches in the context of the spring pendulum problem. Herbert’s treatment of the secondary instability by Floquet theory falls into the latter category. He presents a general formulation of the theory incorporating the spatial growth of a Blasius boundary layer, and he considers detuned modes (in addition to subharmonic and fundamental ones) for combination resonance.

The spatial growth of a boundary layer is one of the key factors in the receptivity problem. The asymptotic analysis of Goldstein clearly establishes this factor as responsible for the interaction between a large-scale

external disturbance and a comparatively small-scale Tollmien-Schlichting wave in the boundary layer. A similar receptivity problem is treated numerically by Gatski and Grosch. They solve the two-dimensional, time-dependent incompressible Navier-Stokes equations by a second-order accurate finite-difference method for the unsteady flow past a semi-infinite flat plate with a sharp leading edge. Their preliminary results demonstrate how an external disturbance can engender instability waves in the leading edge region. Lekoudis studies the quasi-parallel linear spatial stability of the three-dimensional boundary layer on an ellipsoid at incidence, and shows that his results agree qualitatively with experiment.

The nonlinear stability of separating flows such as the leading edge separation bubble, mid-chord or trailing-edge separation, and flows past surface-mounted obstacles, is of practical importance. Smith presents a progress report on his continuing work in this area. This work is unique, addressing questions which are truly nonlinear and nonparallel and holds promise of a unified theory of instability and transition. The article of Bodonyi and Smith is in a similar vein and provides results of practical significance for short-scale Rayleigh instabilities in the flow past surface-mounted obstacles.

Mack's masterly review of linear compressible stability theory is both succinct and complete. He emphasizes those particular aspects which distinguish the compressible from the incompressible theory. He also addresses the possibility of a multiplicity of inviscid and viscous solutions in the compressible case and their possible physical significance. He points out that even the incipient stages of transition at supersonic and hypersonic speeds are not well understood. The work of Bayliss et al. is an attempt to investigate subsonic stability phenomena by means of a full numerical solution of the two-dimensional compressible Navier-Stokes equations. They study the nonlinear growth of unstable waves in a flat plate boundary layer and their control by active heating and cooling of the plate.

The Taylor-Görtler instability mechanism associated with the curved Stokes layers is the focus of Hall's article. He relates this problem to the instability of a fluid layer heated time periodically from below. A similar instability can occur in general flows which are spatially varying and periodic in time. The next three articles deal with classical plane Stokes layers: The paper by von Kerczek discusses the modifications to the stability characteristics resulting from a sinusoidal time modulation of the well-known Hagen-Poiseuille flow, Eckman layer, and the flow of a liquid film down a vertical plate. Monkewitz and Bunster present both experimental and theoretical results on Stokes layers for Reynolds numbers (based on boundary layer thickness and free stream amplitude) up to 600. Their theoretical model is a quasi-steady approach involving nonlinear interactions between modes with a rather heuristic assumption that the difference in phase speeds and growth rates is small. Cowley's approach is based on a multiple scale expansion for asymptotically large Reynolds numbers; this yields to leading order a Rayleigh eigenvalue problem. He

concludes that high-frequency perturbations to a Stokes layer can grow and that the most significant growth rates tend to occur as the outer flow is decelerating. This agrees qualitatively with experimental observation.

The Landau-Stuart-Watson equation has been crucial to many nonlinear stability theories. It has explained many instability phenomena of physical interest. Recently, it has become a subject of study in its own right, being a prototypical equation giving rise to bifurcation and low-order spatial chaos. Sirovich and Newton have devised a clever combination of analysis and numerics which yields an efficient method for studying its stability and bifurcation properties over a wide parameter range. Their article gives a flavor of their recent investigations.

Full numerical solutions of the Navier-Stokes equations are playing an increasingly important role in the study of stability and transition to turbulence. The next two papers are typical of such studies: Ghaddar and Patera's study of stability and resonance in grooved-channel flows is both original and systematic. Streett and Hussaini's article is a preliminary demonstration of work in progress pertaining to the Taylor experiment on Couette flow between concentric rotating cylinders with fixed end walls.

Last, but not the least, is the experimental investigation by Williams of the vortical structures in the breakdown stage of transition in a flat plate boundary layer. Such experiments supplemented by simulation are crucial in unravelling the delicate mechanisms involved in the transition process.

Finally, the editors would like to take this opportunity to thank the participants in the workshop for their contributions, and the staff of Springer-Verlag for their assistance in putting together this volume.

DLD, MYH

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