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Jack K. Hale Sjoerd M. Verduyn Lunel

Introduction to Functional Differential Equations

With 10 Illustrations



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Jack Hale
School of Mathematics
Georgia Institute of
Technology
Atlanta, GA 30332
USA

Sjoerd M. Verduyn Lunel
Vrije Universiteit
Amsterdam
De Boelelaan 1081a
1081 HV Amsterdam
The Netherlands

Editors

F. John
Courant Institute of
Mathematical Sciences
New York University
New York, NY 10012
USA

J.E. Marsden
Department of
Mathematics
University of California
Berkeley, CA 94720
USA

L. Sirovich
Division of
Applied Mathematics
Brown University
Providence, RI 02912
USA

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Preface

The present book builds upon an earlier work of J. Hale, “Theory of Functional Differential Equations” published in 1977. We have tried to maintain the spirit of that book and have retained approximately one-third of the material intact. One major change was a complete new presentation of linear systems (Chapters 6–9) for retarded and neutral functional differential equations. The theory of dissipative systems (Chapter 4) and global attractors was completely revamped as well as the invariant manifold theory (Chapter 10) near equilibrium points and periodic orbits. A more complete theory of neutral equations is presented (see Chapters 1, 2, 3, 9, and 10). Chapter 12 is completely new and contains a guide to active topics of research. In the sections on supplementary remarks, we have included many references to recent literature, but, of course, not nearly all, because the subject is so extensive.

Jack K. Hale
Sjoerd M. Verduyn Lunel

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