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FOREWORD

This IMA Volume in Mathematics and its Applications

LINEAR ALGEBRA FOR SIGNAL PROCESSING

is based on the proceedings of a workshop that was an integral part of the 1991–92 IMA program on “APPLIED LINEAR ALGEBRA.”

We thank Adam Bojanczyk and George Cybenko for organizing the workshop and for editing the proceedings. We also take this opportunity to thank Air Force Office of Scientific Research, National Security Agency, and the National Science Foundation, whose financial support made the workshop possible.

Avner Friedman

Willard Miller, Jr.

PREFACE

This volume contains papers based on lectures presented at the IMA Workshop on Linear Algebra for Signal Processing held at the Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, April 6–10, 1992.

Signal processing applications have burgeoned this past decade. During that same time, signal processing techniques have matured rapidly and now include tools from many areas of mathematics, computer science, physics and engineering. This trend will continue as many new signal processing applications are opening up in consumer products and communications systems.

In particular, signal processing has been making increasingly sophisticated use of linear algebra on both theoretical and algorithmic fronts. Accordingly, the purpose of this workshop was to bring signal processing engineers, computer engineers, and applied linear algebraists together for an exchange of problems, theories and techniques. Particular emphasis was given to exposing broader contexts of the signal processing problems so that the impact of algorithms and hardware could be better understood.

The workshop explored five areas by having a sequence of talks devoted to the underlying signal processing problem, the algorithmic and analytic techniques and, finally, implementation issues for each area. The five areas were:

- (1) updating SVD and eigendecompositions;
- (2) adaptive filtering;
- (3) structured matrix problems;
- (4) wavelets and multirate signal processing; and
- (5) linear algebra architectures (parallel/vector and other high performance machines/designs).

The workshop was attended by a group of excellent researchers. Many innovative concepts were presented in lectures as well as during less formal discussions. An account of these is given by this collection of papers.

The support of the Institute Mathematics and its Applications, University of Minnesota, which is gratefully acknowledged, contributed to the success of the meeting.

Adam Bojanczyk and George Cybenko

CONTENTS

Foreword	xi
Preface	xiii
Structured matrices and inverses	1
<i>P. Comon</i>	
Structured condition numbers for linear matrix structures	17
<i>I. Gohberg and I. Koltracht</i>	
The canonical correlations of matrix pairs and their numerical computation	27
<i>Gene H. Golub and Hongyuan Zha</i>	
Continuity of the joint spectral radius: Application to wavelets	51
<i>Christopher Heil and Gilbert Strang</i>	
Inversion of generalized Cauchy matrices and other classes of structured matrices	63
<i>Georg Heinig</i>	
Wavelets, filter banks, and arbitrary tilings of the time-frequency plane ..	83
<i>C. Herley, J. Kovačević, and M. Vetterli</i>	
Systolic algorithms for adaptive signal processing	125
<i>Marc Moonen</i>	
Adaptive algorithms for blind channel equalization	139
<i>John G. Proakis</i>	
Square-root algorithms for structured matrices, interpolation, and completion problems	153
<i>A.H. Sayed, T. Constantinescu, and T. Kailath</i>	