Multidimensional Analysis

George W. Hart

Multidimensional Analysis

Algebras and Systems for
Science and Engineering

With 19 illustrations

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George W. Hart
Department of Electrical Engineering
Columbia University
New York, NY 10027
USA

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For Carol,

and for Christopher, Colin, Victoria, and the rest of the next generation...

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