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Vibration of Discrete and Continuous Systems

Second Edition

With 147 Figures

Springer

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Series Preface

Mechanical engineering, an engineering discipline borne of the needs of the industrial revolution, is once again asked to do its substantial share in the call for industrial renewal. The general call is urgent as we face profound issues of productivity and competitiveness that require engineering solutions, among others. The Mechanical Engineering Series features graduate texts and research monographs intended to address the need for information in contemporary areas of mechanical engineering.

The series is conceived as a comprehensive one that covers a broad range of concentrations important to mechanical engineering graduate education and research. We are fortunate to have a distinguished roster of consulting editors on the advisory board, each an expert in one of the areas of concentration. The names of the consulting editors are listed on the next page of this volume. The areas of concentration are: applied mechanics; bio-mechanics; computational mechanics; dynamic systems and control; energetics; mechanics of materials; processing; thermal science; and tribology.

Professor Marshek, the consulting editor for dynamic systems and control, and I are pleased to present the second edition of *Vibration of Discrete and Continuous Systems* by Professor Shabana. We note that this is the second of two volumes. The first deals with the *theory of vibration*.

Austin, Texas

Frederick F. Ling

Preface

The theory of vibration of single and two degree of freedom systems is covered in the first volume of this book. In the treatment presented in the first volume, the author assumed only a basic knowledge of mathematics and dynamics on the part of the student. Therefore, the first volume can serve as a textbook for a first one-semester undergraduate course on the theory of vibration. The second volume contains material for a one-semester graduate course that covers the theory of multi-degree of freedom and continuous systems. An introduction to the finite-element method is also presented in this volume. In the first and the second volumes, the author attempts to cover only the basic elements of the theory of vibration that students should learn before taking more advanced courses on this subject. Each volume, however, represents a separate entity and can be used without reference to the other. This gives the instructor the flexibility of using one of these volumes with other books in a sequence of two courses on the theory of vibration. For this volume to serve as an independent text, several sections from the first volume are used in Chapters 1 and 5 of this book.

SECOND EDITION

Several important additions and corrections have been made in the second edition of the book. Several new examples also have been provided in several sections. The most important additions in this new edition can be summarized as follows: Three new sections are included in Chapter 1 in order to review some of the basic concepts used in dynamics and in order to demonstrate the assumptions used to obtain the single degree of freedom linear model from the more general multi-degree of freedom nonlinear model. Section 3.7, which discusses the case of proportional damping in multi-degree of freedom systems has been significantly modified in order to provide a detailed discussion on *experimental modal analysis techniques* which are widely used in the vibration analysis of complex structural and mechanical systems. A new section, Section 5.10, has been introduced in order to demonstrate the use of the finite-element

method in the large rotation and deformation analysis of mechanical and structural systems. The *absolute nodal coordinate formulation*, described in this section, can be used to efficiently solve many vibration problems such as the vibrations of cables and flexible space antennas. A new chapter, Chapter 6, was added to provide a discussion on the subject of *similarity transformation* which is important in understanding the numerical methods used in the large scale computations of the eigenvalue problem. In this new chapter, the Jacobi method and the **QR** decomposition method, which are used to determine the natural frequencies and mode shapes, are also discussed.

CONTENTS OF THE BOOK

The book contains six chapters and an appendix. In the appendix, some of the basic operations in vector and matrix algebra, which are repeatedly used in this book, are reviewed. The contents of the chapters can be summarized as follows:

Chapter 1 of this volume covers some of the basic concepts and definitions used in dynamics, in general, and in the analysis of single degree of freedom systems, in particular. These concepts and definitions are also of fundamental importance in the vibration analysis of multi-degree of freedom and continuous systems. Chapter 1 is of an introductory nature and can serve to review the materials covered in the first volume of this book.

In Chapter 2, a brief introduction to Lagrangian dynamics is presented. The concepts of generalized coordinates, virtual work, and generalized forces are first introduced. Using these concepts, Lagrange's equation of motion is then derived for multi-degree of freedom systems in terms of scalar energy and work quantities. The kinetic and strain energy expressions for vibratory systems are also presented in a matrix form. Hamilton's principle is discussed in Section 6 of this chapter, while general energy conservation theorems are presented in Section 7. Chapter 2 is concluded with a discussion on the use of the principle of virtual work in dynamics.

Matrix methods for the vibration analysis of multi-degree of freedom systems are presented in Chapter 3. The use of both Newton's second law and Lagrange's equation of motion for deriving the equations of motion of multi-degree of freedom systems is demonstrated. Applications related to angular oscillations and torsional vibrations are provided. Undamped free vibration is first presented, and the orthogonality of the mode shapes is discussed. Forced vibration of the undamped multi-degree of freedom systems is discussed in Section 6. The vibration of viscously damped multi-degree of freedom systems using proportional damping is examined in Section 7, and the case of general viscous damping is presented in Section 8. Coordinate reduction methods using the modal transformation are discussed in Section 9. Numerical methods for determining the mode shapes and natural frequencies are discussed in Sections 10 and 11.

Chapter 4 deals with the vibration of continuous systems. Free and forced vibrations of continuous systems are discussed. The analysis of longitudinal, torsional, and transverse vibrations of continuous systems is presented. The orthogonality relationships of the mode shapes are developed and are used to define the modal mass and stiffness coefficients. The use of both elementary dynamic equilibrium conditions and Lagrange's equations in deriving the equations of motion of continuous systems is demonstrated. The use of approximation methods as a means of reducing the number of coordinates of continuous systems to a finite set is also examined in this chapter.

In Chapter 5 an introduction to the finite-element method is presented. The assumed displacement field, connectivity between elements, and the formulation of the mass and stiffness matrices using the finite-element method are discussed. The procedure for assembling the element matrices in order to obtain the structure equations of motion is outlined. The convergence of the finite-element solution is examined, and the use of higher order and spatial elements in the vibration analysis of structural systems is demonstrated. This chapter is concluded with a discussion on the use of the *absolute nodal coordinate formulation* in the large finite-element rotation and deformation analysis.

Chapter 6 is devoted to the eigenvalue analysis and to a more detailed discussion on the similarity transformation. The results presented in this chapter can be used to determine whether or not a matrix has a complete set of independent eigenvectors associated with repeated eigenvalues. The definition of Jordan matrices and the concept of the generalized eigenvectors are introduced, and several computer methods for solving the eigenvalue problem are presented.

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Chicago, Illinois

Ahmed A. Shabana

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