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Ole E. Barndorff-Nielsen

Parametric Statistical Models
and Likelihood

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Preface

This book is a slightly revised and expanded version of a set of notes used for a lecture series given at the École d'Été de Probabilités at St. Flour in August 1986. In view of the statistical nature of the material discussed herein it was agreed to publish the material as a separate volume in the Statistics Series rather than, as is the tradition, in a joint volume in the Lecture Notes in Mathematics Series.

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O.E. Barndorff-Nielsen

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0. Introduction

0.1. Outline of contents	1
0.2. A few preliminaries	2

1. Likelihood and auxiliary statistics

1.1. Likelihood	4
1.2. Moments and cumulants of log likelihood derivatives	10
1.3. Parametrization invariance	13
1.4. Marginal and conditional likelihood	15
1.5. Combinants, auxiliaries, and the p^* -model	19
1.6. Orthogonal parameters	27
1.7. Pseudo likelihood, profile likelihood and modified profile likelihood	30
1.8. Ancillarity and conditionality	33
1.9. Partial sufficiency and partial ancillarity	41
1.10. Likelihood expansions	43
1.11. Additional bibliographical notes	47

2. Transformation models and exponential models

2.1. Group actions and invariant measures	49
2.2. Transformation models	62
2.3. Transformation submodels	83

2.4.	Exponential models	87
2.5.	Exponential transformation models	98
2.6.	Additional bibliographical notes	102

3. Reparametrizations and differential geometry

3.1.	Multiarrays	104
3.2.	Tensors and affine connections	107
3.3.	Strings	117
3.4.	Covariant differentiation and strings	123
3.5.	Intertwining	126
3.6.	Submanifolds	137
3.7.	Geometric measures	139
3.8.	Manifolds with a Lie group action	140
3.9.	Fibre bundles, connections and (parallel) transport	146
3.10.	Additional bibliographical notes.	151

4. Inferential and geometric structures

4.1.	Ancillary statistics and conditionality structures	152
4.2.	Conditionality structures for transformation models	153
4.3.	Construction of approximately ancillary statistics	153
4.4.	Jacobians of conditionality structures	156
4.5.	Geometry of parametric models	158
4.6.	Additional bibliographical notes	172

5. Cumulants

5.1.	Elemental properties of cumulants	173
5.2.	Relations between moments and cumulants	177
5.3.	An alternative definition of generalized cumulants	186
5.4.	Additional bibliographical notes	187

<u>6. Laplace's method. Edgeworth and saddle-point approximations</u>	
6.1. Laplace's method	188
6.2. Hermite polynomials	191
6.3. Edgeworth approximations	199
6.4. Saddle-point approximations	207
6.5. Additional bibliographical notes	211
 <u>7. Distributions of likelihood quantities</u>	
7.1. The distribution of the maximum likelihood estimator	213
7.2. Expansion of p^*	224
7.3. The distribution of the score vector	229
7.4. The distribution of likelihood ratio statistics	230
7.5. Modified profile likelihood	235
7.6. Additional bibliographical notes	249
 <u>Appendices</u>	
A.1. Taylor's formula	250
A.2. Fourier transformation	250
A.3. Some formulas for matrices and determinants	251
A.4. Partially ordered sets, partitions and Möbius inversion	253
A.5. The Legendre transform	257
A.6. A differential geometric inversion result	260
 <u>References</u>	262
 <u>Subject index</u>	268