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Preface

This is the first of two volumes representing the current state of knowledge about Enriques surfaces which occupy one of the classes in the classification of algebraic surfaces. Recent improvements in our understanding of algebraic surfaces over fields of positive characteristic allowed us to approach the subject from a completely geometric point of view although heavily relying on algebraic methods.

Some of the techniques presented in this book can be applied to the study of algebraic surfaces of other types. We hope that it will make this book of particular interest to a wider range of research mathematicians and graduate students.

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