Hüseyin Koçak

Differential and Difference Equations through Computer Experiments

With Diskettes Containing PHASER: An Animator/Simulator for Dynamical Systems for IBM Personal Computers

Second Edition

With 108 Illustrations

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To my parents:

Hatice and Irfan,
even though they cannot read these words.

Preface

This is a somewhat unusual book with a dual purpose. First, it is a manual to help readers learn how to use PHASER, the program on the accompanying diskette for IBM personal computers. Second, it is an illustrated guide to the wonderful world of experimental and theoretical dynamics, one which presents dozens of concrete examples ranging from the most rudimentary, appropriate for the beginning student, to the highly complex, suitable for the research mathematician.

Before indicating what PHASER does and how it works, let me describe how it came about. During the past decade the field of differential and difference equations has witnessed a remarkable explosion of knowledge, not only in theory but also in applications to disciplines as diverse as biology and fluid mechanics. Computers have played a crucial role in this process by making possible detailed analyses of specific systems. In this regard, one need only mention the work of Lorenz on strange attractors and the discoveries of Feigenbaum on the bifurcations of interval maps.

It was with the intention of bringing some of this excitement to Brown undergraduates that I began to develop, about three years ago, a new course in the Division of Applied Mathematics bearing the same title as this book. I decided to collaborate with several mathematically-oriented computer science students on the design and the implementation of software for the course in order to take advantage of recent advances in computer-aided instruction by the Department of Computer Science. My main goal was to create a sophisticated interactive simulator for difference and differential equations, one that did not require the user to have any programming knowledge. I also wanted the program to provide the necessary tools for live demonstrations during lectures, and for experimentation and research by students and

faculty in Applied Mathematics. After three years, five master's students, and endless revisions, PHASER, and eventually this book, came into being.

Now, let me briefly describe how PHASER works and what it does. It is an extremely versatile and easy-to-use program, incorporating state-of-the-art software technology (menus, windows, etc.) in its user interface. The user first creates, with the help of a menu, a suitable window configuration for displaying a combination of views -- phase portraits, texts of equations, Poincare sections, etc. Next, the user can specify, from another menu, various choices in preparation for numerical computations. He or she can choose, for instance, to study from a library of many dozen equations, and then compute solutions of these equations with different initial conditions or step sizes, while interactively changing parameters in the equations. From yet another menu, these solutions can be manipulated graphically. For example, the user can rotate the images, take sections, etc. During simulations, the solutions can be saved in various ways: as a hardcopy image of the screen, as a printed list, or in a form that can be reloaded into PHASER at a later time for demonstrations or further work.

Since its initial implementation in our computing laboratory two years ago, PHASER has been used in conjunction with our beginning and advanced courses as a means of improving applied mathematics pedagogy through an example-oriented, "hands-on" approach. I am happy to be able to say that PHASER has been well received by students and faculty alike at Brown University.

Despite the success of PHASER, I cannot help including a word of caution in this preface. The subject of dynamical systems is vast and colorful, but also inherently difficult. The danger of oversimplification is quite real, and it is easy to get the false impression that computers can provide answers to all questions. The user should always remain aware that numerical simulations have the potential to be misleading. PHASER is not a substitute for the theory of dynamical systems; rather, it is meant to be a complementary tool for performing mathematical experiments and for illustrating the theory with concrete examples. Whatever the purpose for which it is utilized, it is my hope that PHASER will help the subject of ordinary differential and difference equations come alive for its users.

Huseyin Kocak
Brown University
August 1985

Preface to the Second Printing/Edition

Although this constitutes the second printing of *Differential and Difference Equations through Computer Experiments*, the accompanying program is decidedly the second edition, or version 1.1, of PHASER. While the design of the program remains essentially unchanged, it now has the capacity to take advantage of the higher resolution EGA or VGA graphics on IBM Personal Computers. For those who have only CGA graphics, the original version of PHASER is also included. This low resolution version has now been modified to run on EGA/VGA boards as well, primarily for use with inexpensive LCD projectors. For further information regarding installation on a hard disk, printer support, etc., please read the **Read-Me Chapter**, and the **readme** file on the diskette.

I wish to express my gratitude to many users from all continents who have written to me about both PHASER and the text. Your suggestions and critical observations have been most helpful. I only regret not being able to respond to all of you in detail. However, some of the most common requests—a simpler user-interface for entering new equations, a new algorithm (Shampine-Gordon) for stiff systems, and Liapunov exponents, unstable manifolds, basin boundaries, etc.—are slated for inclusion in the projected second edition of the book.

Finally, I would like to thank Springer-Verlag for obtaining a licence to use the HALO graphics library for the development of PHASER.

Hüseyin Koçak
Providence, August 1988

Read-Me Chapter

In this preliminary chapter, we discuss hardware requirements, the contents of chapters, and how to use this book most profitably. As the title suggests, it is required reading for everyone who may wish to use PHASER in instruction or in research.

Whom is PHASER for ?

Modesty aside, *PHASER: An Animator/Simulator for Dynamical Systems* is for all, from freshmen to researchers, who are remotely interested in difference or ordinary differential equations. This book and diskette combination makes a unique complement to the standard textbook approach by allowing the user to analyze specific equations of theoretical and practical importance, and to gain insight into their *dynamics.* In fact, PHASER has been used in the Division of Applied Mathematics, Brown University, both for an elementary course in conjunction with Boyce & DiPrima [1977], and for a more advanced senior/graduate course. The enthusiastic response of our students, above all their return on their own to the computing laboratory to satisfy newly aroused curiosity, has been most pleasing. The sophisticated interactive graphical capabilities of PHASER make it a useful exploratory tool for researchers as well.

Hardware requirements

To run PHASER, you need the following options on your IBM Personal Computer PC, XT, AT, or PS/2:

- DOS Version 2.0 (or higher) operating system,

- 256K-bytes of memory,
- IBM Color Graphics Adapter (CGA), or Enhanced Graphics Adapter (EGA), or Video Graphics Adapter (VGA),
- Color Display. A mono-chrome monitor will work; however, some colors (such as red) may be invisible.
- One version of PHASER supports the Mathematics Coprocessor. We do recommend this hardware option because the gains in computational speeds, especially on a PC or XT, are considerable. The version that requires the Mathematics Coprocessor will not run on a machine that does not have it.

The diskettes

The 5-1/4" diskette in the jacket on the back inside cover is formatted at 1.2M. In the interest of keeping user costs to a minimum, this diskette has been formatted at high density. Those with low density 5-1/4" diskette drives alone will need to transfer the appropriate copy of PHASER to a low density diskette. The two 3-1/2" diskettes, one for EGA/VGA and the other for CGA, are formatted at low density, 720K. The combined contents of these two diskettes are identical to that of the 5-1/4" one.

For safety, make sure the original diskette is write-protected and then make a back-up copy. If you have a hard disk, simply transfer the desired copy of PHASER to the disk, as explained below.

Installation on hard disk

To install PHASER on your hard disk, first make a directory of your choice and change to that directory. Then, insert the appropriate diskette in drive A, and, depending on your hardware configuration, type one of the following install batch commands:

`a:install1` for EGA/VGA graphics WITHOUT Math. Coprocessor
`a:install2` for EGA/VGA graphics WITH Math. Coprocessor
`a:install3` for CGA graphics WITHOUT Math. Coprocessor
`a:install4` for CGA graphics WITH Math. Coprocessor

For example, to install PHASER in a directory named *phaser* on a machine with EGA or VGA graphics and Mathematics Coprocessor, type the following three commands:

```
C:> mkdir phaser      < ENTER >
C:> cd phaser         < ENTER >
```

`C:\phaser> a:install2` *< ENTER >*

Now, PHASER is installed in a directory called *phaser*. To run the program, simply type, while in your directory *phaser*:

`C:\phaser> phaser` *< ENTER >*

About ten seconds after you type **phaser**, a logo will appear, followed by the screen image in *Figure 4.2*, and you will be ready to begin simulations.

If you prefer to install PHASER manually, here are the files for EGA or VGA: **phaser.exe** for EGA or VGA graphics WITH Mathematics Coprocessor; **nphaser.exe** for EGA or VGA graphics WITHOUT Mathematics Coprocessor. Copy one of these and the file **demo**. Create a subdirectory named **files** and copy everything in the **files** into it.

All the executables and the supporting files for CGA are in a subdirectory called **cga**. Change to this directory and copy one of the executable files—**phaser.exe** for CGA graphics WITH Mathematics Coprocessor, or **nphaser.exe** for CGA graphics WITHOUT Mathematics Coprocessor—and the remaining files, including everything in the subdirectory **files**.

If you already have PHASER in your computer, copy the desired new executable and the new **files** subdirectory over the existing ones. In this way, you can use your own previously user-entered equations.

Printer support

If you are using the CGA version of PHASER, before starting the program, you should first execute the DOS graphics function by typing

graphics *< ENTER >*

This function allows you to print the contents of the screen on the graphics printer at any time during simulations. This is done by pressing the special <Print Screen>, while holding the <Shift> key.

Unfortunately, the DOS facility <Shift> <Print Screen> does not work with EGA (or VGA) resolution. To remedy this shortcoming of DOS, the current EGA/VGA version of PHASER supports screen dumps for two of the most common printers:

EPSON/IBM dot matrix and *HP LaserJet+* laser printer.

More specifically, when you wish to send the screen to your printer, use the appropriate entry (**E**pson/IBM or **H**P LaserJet+) on the UTILITIES menu. Printing takes several minutes. The aspect ratio of the screen is different from those of the printers; to get the 'correct' aspect ratio on your printer, adjust the *Windosize* from the NUMERICS Menu. In the future,

a larger selection of printers will be supported. In the meantime, if you do not have one of these two printers, you may wish to obtain a commercially available memory-resident program for screen dumps.

Notes

- The aspect ratio of the EGA screen is about 14/10; therefore, the default *Windosize* is now -14, 14, -10, 10.
- A new menu entry "o" (oh, not zero) is added to the NUMERICS menu for computing orbits in both forward and backward time. This entry does not work with difference equations.
- The figures in Chapter 4 and 5 are made at the EGA resolution. All other figures are at the CGA resolution.
- With user-defined second-order time-periodic systems, the Poincaré map gives only one point, because the solutions are in $\mathbf{R}^3$, not on a cylinder. For example, with a user-defined equation, it is not possible to reproduce *Figure 7.20*. This will be remedied in the next version.

Acknowledgments

Many students and colleagues contributed a great deal to both the conception and the realization of PHASER. First and foremost, I would like express my heartfelt thanks to my students Lisa Heavey, Matthew Merzbacher, Manijeh Shayegan, Mark Sommer, and Michael Strickman. They spent endless hours, both day and night, writing almost all of the code for PHASER through its many versions. Without their energetic collaborations, PHASER would have forever remained only an idea.

Stepping back in time, my graduate school mentors Al Kelley and Ralph Abraham persuaded me that experimental dynamics could be an enjoyable and indispensable complement to classical mathematics. Later at Brown, the graphical vision and enthusiasm of Andries van Dam, as well as his material support, gave me the courage to blend mathematics with computer science. My adventures into higher dimensions with Thomas Banchoff, Frederic Bisshopp, David Laidlaw, and David Margolis were equally invaluable in sharpening my skills and heightening my appreciation of computer graphics. The Lefschetz Center for Dynamical Systems and the Division of Applied Mathematics of Brown University, by providing the necessary care-free, yet stimulating, working environment, enabled me to toy with nonstandard pursuits. In particular, Wendell Fleming encouraged me to design a new course, and Lawrence Sirovich stopped by every Monday afternoon to ask if the book were done.

On the mathematical side, I have been most fortunate, during the past three years, to share the same hallway and lunch table with Jack Hale. His expert knowledge and friendship have had a profound effect on me, as well as on PHASER. My association with Donald Aronson has likewise been critical in shaping the project. I have, in addition, received mathematical, material, or moral support from Shui-Nee Chow, John Guckenheimer, Philip Holmes, John Mallet-Paret, Donald McClure, Susan Schmidt, Thomas Sharp, and Jorge Sotomayor.

In the final stages of the manuscript, I have also benefitted from the assistance of Philip Davis, Susan McGowen, Wayne Nagata, Brian Weibel, and Andy Young. Above all, however, Lee Zia and Nancy Lawther not only read the entire book, but also made sure that I enjoyed the difficult task of writing and rewriting.

Thank you all; I could not have done it without you.

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