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Algebraic Aspects
of Integrable Systems
In Memory of Irene Dorfman

A. S. Fokas and I. M. Gelfand
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Preface

Irene Dorfman died in Moscow on April 6, 1994, shortly after seeing her beautiful book on Dirac structures [1]. The present volume contains a collection of papers aiming at celebrating her outstanding contributions to mathematics. Her most important discoveries are associated with the algebraic structures arising in the study of integrable equations. Most of the articles contained in this volume are in the same spirit.

Irene, working as a student of Israel Gel'fand made the fundamental discovery that integrability is closely related to the existence of bi-Hamiltonian structures [2], [3]. These structures were discovered independently, and almost simultaneously, by Magri [4] (see also [5]). Several papers in this book are based on this remarkable discovery. In particular Fokas, Olver, Rosenau construct large classes on integrable equations using bi-Hamiltonian structures, Fordy, Harris derive such structures by considering the restriction of isospectral flows to stationary manifolds and Fuchssteiner discusses similar structures in a rather abstract setting.

Bi-Hamiltonian operators and their related Poisson brackets can be constructed via the so called Adler-Gelfand-Dickey [6], [7] approach. This construction is based on Lax operators [8] which are regarded as elements of the algebra of pseudo-differential symbols. Irene made important contributions in this direction (see for example [9]). Several papers in this book use extensively the algebra of pseudo-differential operators. In particular Oevel discusses such operators in connection with r -matrices and the modified Yang-Baxter equation, Dickey constructs the τ -function for large classes of systems of integrable equations, and Cohen, Manin, Zagier establish some beautiful relations between certain pseudo-differential operators and modular forms.

In recent years there has been much activity aiming at extending some of the above results from continuous to discrete systems. Several discrete versions of the equation $u_t + uu_x = 0$, of certain "Schwarzian" equations, and of the Newmann equation are discussed in the papers of Kuper-Shmidt, of Nijhoff, and of Ragnisco, Suris respectively. Nonultralocal Poisson brackets for 1-dimensional lattice systems are discussed by Semenov-Tian-Shansky and Sevostyanov.

Irene dedicated a considerable part of her research studying the concept of mastersymmetries introduced in [10] (see chapter [7] of her book). This concept is used by Grunbaum and Haine to establish certain relations

between the Toda lattice, Bocher's theorem, and orthogonal polynomials.

The study of the algebraic properties of integrable equations, and in particular the existence of infinitely many symmetries for such equations, has been used to study equations which are integrable only asymptotically (with respect to some small parameter ϵ). Two different approaches to asymptotic integrability are presented in the papers of Kodama, Mikhailov and of Santini.

Ferapontov and Mokhov use certain transformations to map the Witten-Dijkgraaf-Verlinde-Verlinde equations (i.e. the associativity equations) to a system of integrable equations of the hydrodynamic type. Schief and Rogers show that the $2 + 1$ Loewner-Konopelchenko-Rogers system is invariant under certain Laplace-Darboux transformations and then use this fact to obtain interesting integrable reductions.

Although most papers presented here are algebraic, some papers have a strong analytic flavor. In particular McKean uses some beautiful trace formulae associated with the Dirac and the Schrödinger periodic operators to study the canonical 1-forms of the symplectic geometry associated with the defocussing nonlinear Schrödinger and with the Korteweg-deVries equations. Alber, Luther and Marsden obtain interesting new classes of solutions of nonlinear PDE's by constructing certain finite dimensional integrable Hamiltonian systems on Riemannian manifolds.

A.S. Fokas and I. M. Gelfand, Editors

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*This volume is dedicated to the memory
of our dear friend and colleague
Irene Dorfman*