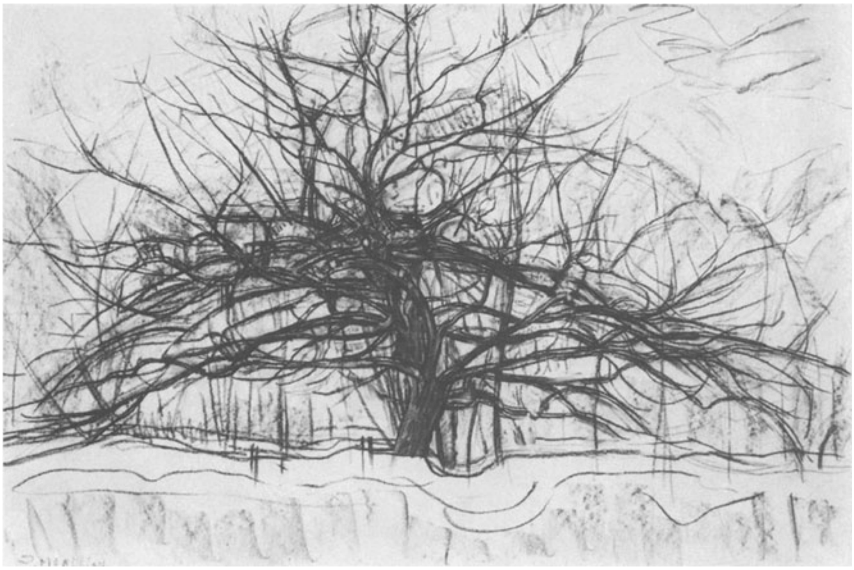

Piet Mondrian "Tree," 1913.

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Hyman Bass
Alexander Lubotzky

Tree Lattices

with Appendices by
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Dedicated to our roots and branches

in memory of my grandmother

טובה בליזובסקי

A.L.

with love to my children,

Anne, Ivan and Gabriella

H.B.

and in admiring tribute to

J.-P. Serre,

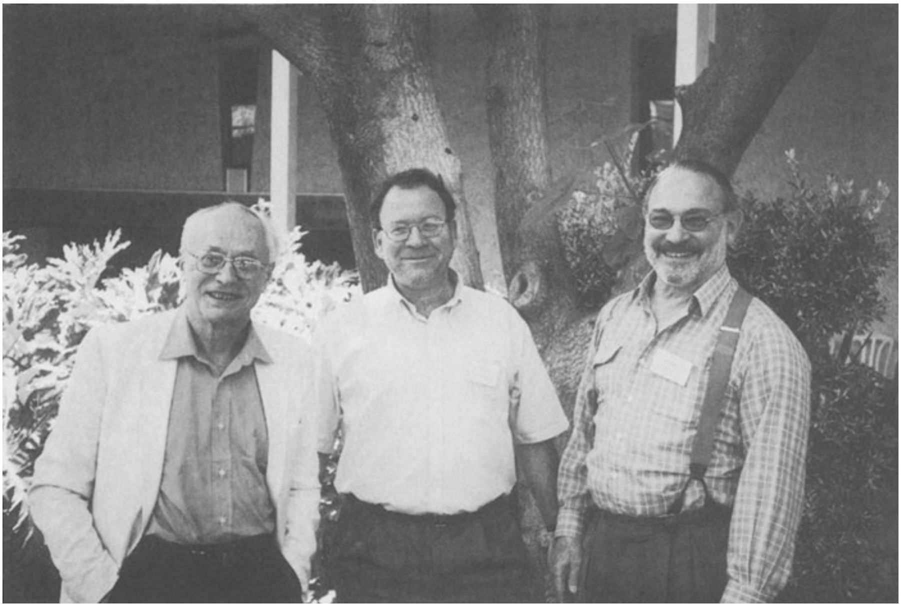

Photo by Lisa Carbone, Jerusalem, Israel, May 2000

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Preface

Let H be a non compact simple Lie group. In most cases H can be realized as essentially the full isometry group of some geometry X, where X is a contractible metric space. This geometric action is the natural tool for studying H and its discrete subgroups, in particular its lattices.

In the case of real or complex Lie groups, X is the symmetric space, a Riemannian manifold. In case H is a Lie group over a non archimedian (for example a p-adic) field, X is the Bruhat–Tits building, a contractible simplicial complex of dimension equal to the rank of H.

There is one notable exception to the above picture, namely when H is non archimedean of rank one, in which case X is a bi-regular tree. In this case the full isometry group $G = \text{Aut}(X)$ is vastly larger than its subgroup H. Nonetheless Tits has shown that G itself is a virtually simple locally compact group, and one can naturally study its discrete subgroups and lattices. That is one purpose of the present work.

More generally we study the automorphism group $G = \text{Aut}(X)$ of any locally finite tree X, and its discrete subgroups Γ. We call Γ an X-lattice if the volume of $X \text{mod} \Gamma$, suitably defined, is finite. These are the tree lattices of the book's title. We present a fairly systematic investigation of the existence, structure, and properties of tree lattices, drawing parallels and contrasts, whenever possible with the situation for lattices in Lie groups. We give much attention to the construction of diverse examples. The methods used are based on the notion of graphs of groups, as first developed in the book *Trees* of Serre.

While the theory of lattices in Lie groups motivates many of the questions investigated here, we do not rely on that theory for the present work, which is essentially elementary and self contained.

Many of the results of the book appear here for the first time in print. This work has evolved slowly over about a decade. Some of the main results presented or cited appeared first in early drafts as conjectures. There are three appendices. One describes a group theoretic construction provided by Peter Neumann which we use here for producing certain self normalizing non uniform tree lattices. A second appendix, by one of us with J. Tits, presents a criterion for the full automorphism group of a tree to be discrete. The third, by one of us with L. Carbone and G. Rosenberg, presents the proof that, whenever G is unimodular and $G\backslash X$ has finite volume, there exist X-lattices. Initially this was only conjectured, and proved in special cases. We are grateful to the other authors for permission to publish these results here.

Finally, we owe a great debt to Ann Kostant and Elizabeth Loew for generous and skillful editorial and production assistance in bringing this manuscript to final completion.

Hyman Bass
Alexander Lubotzky
Jerusalem, July, 2000