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Invariant Manifolds and Fibrations for Perturbed Nonlinear Schrödinger Equations

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Preface

This book presents a development of invariant manifold theory for a specific canonical nonlinear wave system – the perturbed nonlinear Schrödinger equation. The main results fall into two parts. The first part is concerned with the persistence and smoothness of locally invariant manifolds. The second part is concerned with fibrations of the stable and unstable manifolds of inflowing and overflowing invariant manifolds. The central technique for proving these results is Hadamard's graph transform method generalized to an infinite-dimensional setting. However, our setting is somewhat different than other approaches to infinite dimensional invariant manifolds since for conservative wave equations many of the interesting invariant manifolds are infinite dimensional and noncompact. The style of the book is that of providing very detailed proofs of theorems for a specific infinite dimensional dynamical system-the perturbed nonlinear Schrödinger equation.

The book is organized as follows. Chapter one gives an introduction which surveys the state of the art of invariant manifold theory for infinite dimensional dynamical systems. Chapter two develops the general setup for the perturbed nonlinear Schrödinger equation. Chapter three gives the proofs of the main results on persistence and smoothness of invariant manifolds. Chapter four gives the proofs of the main results on persistence and smoothness of fibrations of invariant manifolds.

This book is an outgrowth of our work over the past nine years concerning homoclinic chaos in the perturbed nonlinear Schrödinger equation. The theorems in this book provide key building blocks for much of that work. We would like to acknowledge the guiding role that Dave McLaughlin has played in this work. We would like to thank our wives, Sherry and Meredith, for their support during the production of this work. Finally, we would like to thank Holly Domingo for her expert preparation of the manuscript. This work has benefited from the support of the National Science Foundation and the Office of Naval Research.

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