The IMA Volumes in Mathematics and its Applications

Volume 110

Series Editor
Willard Miller, Jr.

Springer Science+Business Media, LLC

Institute for Mathematics and its Applications IMA

The **Institute for Mathematics and its Applications** was established by a grant from the National Science Foundation to the University of Minnesota in 1982. The IMA seeks to encourage the development and study of fresh mathematical concepts and questions of concern to the other sciences by bringing together mathematicians and scientists from diverse fields in an atmosphere that will stimulate discussion and collaboration.

The IMA Volumes are intended to involve the broader scientific community in this process.

Willard Miller, Jr., Professor and Director

* * * * * * * * * *

IMA ANNUAL PROGRAMS

1982–1983	Statistical and Continuum Approaches to Phase Transition
1983–1984	Mathematical Models for the Economics of Decentralized Resource Allocation
1984–1985	Continuum Physics and Partial Differential Equations
1985–1986	Stochastic Differential Equations and Their Applications
1986–1987	Scientific Computation
1987–1988	Applied Combinatorics
1988–1989	Nonlinear Waves
1989–1990	Dynamical Systems and Their Applications
1990–1991	Phase Transitions and Free Boundaries
1991–1992	Applied Linear Algebra
1992–1993	Control Theory and its Applications
1993–1994	Emerging Applications of Probability
1994–1995	Waves and Scattering
1995–1996	Mathematical Methods in Material Science
1996–1997	Mathematics of High Performance Computing
1997–1998	Emerging Applications of Dynamical Systems
1998–1999	Mathematics in Biology
1999–2000	Reactive Flows and Transport Phenomena
2000–2001	Mathematics in Multi-Media

Continued at the back

Christoph Börgers Frank Natterer
Editors

Computational Radiology and Imaging

Therapy and Diagnostics

With 97 Illustrations

Springer

Christoph Börgers
Department of Mathematics
Tufts University
Medford, MA 02155, USA

Frank Natterer
Institut für Numerische
und instrumentelle Mathematik
Universität Münster
Einsteinstrasse 62
D-48149 Münster, Germany

Series Editor:
Willard Miller, Jr.
Institute for Mathematics and its
Applications
University of Minnesota
Minneapolis, MN 55455, USA

Mathematics Subject Classifications (1991): 35R30, 44A12, 65R30, 92C50, 92C55

Library of Congress Cataloging-in-Publication Data
Börgers, Christoph.
Computational radiology and imaging : therapy and diagnostics / Christoph Börgers, Frank Natterer.
p. cm. — (The IMA volumes in mathematics and its applications ; v. 110)
Includes bibliographical references and index.
ISBN 978-1-4612-7189-5 ISBN 978-1-4612-1550-9 (eBook)
DOI 10.1007/978-1-4612-1550-9
1. Diagnostic imaging—Mathematics Congresses. I. Natterer, F. (Frank), 1941- . II. Title. III. Series.
RC78.7.D53B67 1999
616.07'54'0151—dc21 99-18598

Printed on acid-free paper.

© 1999 Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. in 1999
Softcover reprint of the hardcover 1st edition 1999
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC,.
except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.
Authorization to photocopy items for internal or personal use, or the internal or personal use of specific clients, is granted by Springer Science+Business Media, LLC.,provided that the appropriate fee is paid directly to Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, USA (Telephone: (508) 750-8400), stating the ISBN number, the title of the book, and the first and last page numbers of each article copied. The copyright owner's consent does not include copying for general distribution, promotion, new works, or resale. In these cases, specific written permission must first be obtained from the publisher.

Production managed by A. Orrantia; manufacturing supervised by Jacqui Ashri.
Camera-ready copy prepared by the IMA.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-7189-5

FOREWORD

This IMA Volume in Mathematics and its Applications

COMPUTATIONAL RADIOLOGY AND IMAGING: THERAPY AND DIAGNOSTICS

is based on the proceedings of a workshop with the same title. The workshop was an integral part of the 1996–97 IMA program on "MATHEMATICS IN HIGH-PERFORMANCE COMPUTING."

I would like to thank the scientific organizers: Christoph Börgers of Tufts University, Department of Mathematics and Frank Natterer of Universität Münster, Institut für Numerische und instrumentelle Mathematik for their excellent work as organizers of the meeting and for editing the proceedings.

I also take this opportunity to thank the National Science Foundation (NSF) and the Army Research Office (ARO), whose financial support made the workshop possible.

Willard Miller, Jr., Professor and Director

PREFACE

The articles collected in this volume are based on lectures at the IMA Workshop "Computational Radiology and Imaging: Therapy and Diagnostics," March 17–21, 1997. Introductory articles by the editors and organizers of the workshop have been added. The focus is on inverse problems involving electromagnetic radiation and particle beams, with applications to X-ray tomography, nuclear medicine, near-infrared imaging, microwave imaging, electron microscopy, and radiation therapy planning.

For electromagnetic radiation with short wave lengths (for example X-rays, gamma rays, and near-infrared lasers), the particle model is appropriate, and the radiation transport is described by the linear Boltzmann equation. For larger wave lengths (for example microwaves), the wave model is appropriate, and the radiation transport is described by Maxwell's equations. Most of the articles in this volume either make explicit reference to the linear Boltzmann equation, or are implicitly based on limiting forms or approximate solutions of this equation. For example, X-ray tomography is based on transport without scattering, a limit of the linear Boltzmann equation in which the mean free path tends to infinity. Algorithms for near-infrared imaging are frequently based on diffusion, a limit of the linear Boltzmann equation in which the mean free path tends to zero. In radiation therapy planning, the quantity of greatest interest is the dose, the energy deposited per unit mass, a macroscopic quantity that could in principle be derived from the solution of a boundary value problem for the linear Boltzmann equation. In practice, simpler approximate dose calculation methods are often used. One of the articles in this volume, the one by Colton and Monk on microwave imaging, is based on the wave model.

The workshop deliberately did not focus on forward problems, even though their computational solution is often a very challenging task by itself, and the subject of extensive current research.

We thank the IMA for hosting the workshop with hospitality and efficiency, especially Avner Friedman, who was the director of the IMA at the time of the workshop, and Robert Gulliver, who was its associate director. The workshop was part of a year on high performance computing at the IMA, and we also thank the organizers of this year, especially Mitchell Luskin and Ridgway Scott. We thank the extraordinarily capable and helpful technical staff at the IMA, especially Patricia V. Brick. Above all, we thank the participants in the workshop for making it productive and inspiring.

Christoph Börgers
Frank Natterer
May 1998

CONTENTS