

## **Lecture Notes in Statistics**

Edited by P. Bickel, P. Diggle, S. Fienberg, K. Krickeberg,  
I. Olkin, N. Wermuth, S. Zeger

**145**

**Springer**

*New York*

*Berlin*

*Heidelberg*

*Barcelona*

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*London*

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**Stochastic Population Models**  
**A Compartmental Perspective**



Springer

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Library of Congress Cataloging-in-Publication Data

Matis, James H.  
Stochastic population models : a compartmental perspective / James H. Matis, Thomas R. Kiffe.  
p. cm. -- (Lecture notes in statistics ; 145)  
Includes bibliographical references and index.  
ISBN 0-387-98657-X (soft cover : alk. paper)  
1. Population biology--Mathematical models. 2. Stochastic processes--Mathematical models. I. Kiffe, Thomas. II. Title. III. Lecture notes in statistics (Springer-Verlag) ; v. 145.

QH352 .M383 2000  
577.8'8'015118--dc21

00-030462

CIP data available.

Printed on acid-free paper.

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9 8 7 6 5 4 3 2 1

ISBN 0-387-98657-X Springer-Verlag New York Berlin Heidelberg SPIN 10697401

To Jeanette, Christine, Marianne, Barbara, and Timothy.  
JHM

To Jacqueline, Tasha, Belle, Alex, Rocky, and Bunny.  
TRK

# Preface

This monograph has been heavily influenced by two books. One is Renshaw's [82] work on modeling biological populations in space and time. It was published as we were busily engaged in modeling African bee dispersal, and provided strong affirmation for the stochastic basis for our ecological modeling efforts. The other is the third edition of Jacquez' [28] classic book on compartmental analysis. He reviews stochastic compartmental analysis and utilizes generating functions in this edition to derive many useful results. We interpreted Jacquez' use of generating functions as a message that the day had come for modeling practioners to consider using this powerful approach as a model-building tool. We were inspired by the idea of using generating functions and related methods for two purposes. The first is to integrate seamlessly our previous research centering in stochastic compartmental modeling with our more recent research focusing on stochastic population modeling. The second, related purpose is to present some key research results of practical application in a natural, user-friendly way to the large user communities of compartmental and biological population modelers.

One general goal of this monograph is to make a case for the practical utility of the various stochastic population models. In accordance with this objective, we have chosen to illustrate the various stochastic models, using four primary applications described in Chapter 2. In so doing, this monograph is based largely on our own published work. We take this approach because we are obviously very familiar and can speak with some authority about the practical motivation, utility and shortcomings of each application. We hope that the modeling user communities will find these examples and ensuing discussion useful.

The monograph presents an approach to analyzing the stochastic models which is undoubtedly new to most readers. Our attempt is to keep the general mathematical level equivalent to that in Jacquez [28] and Renshaw [82]. The theoretical development is outlined in Chapters 3 and 9, and the methods are then applied to the models in the body of the subsequent chapters.

Specialized methods outside the scope of those presented in Chapters 3 and 9, as well as detailed extensions, are covered in the chapter appendices.

We acknowledge with pleasure and great gratitude the past and present contributions of our many colleagues and collaborators. John Jacquez, Eric Renshaw and Qi Zheng gave valuable comments and encouragement at early stages of this work. Among our other esteemed colleagues are Mel Carter, H. O. Hartley and Aldo Rescigno in compartmental analysis; David Allen, Tom Wehrly and P. R. Parthasarathy in stochastic modeling and statistical theory; and W. C. Ellis, Bill Rubink, Bill Grant, Bernie Patten, Gard Otis and Rob Hengeveld in scientific applications. Most of their names occur often in the references, where their contributions are apparent. We are most grateful to them, and to our many students over the years. Finally, this monograph would not have been possible without the skill and dedicated efforts of Yvonne Clark, our technical secretary and advisor, and we thank her for her many years of service. At the same time, we assume complete responsibility for all possible errors and shortcomings.

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