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A. Majda

**Compressible Fluid Flow
and Systems of
Conservation Laws in
Several Space Variables**



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Andrew Majda

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