

Undergraduate Texts in Mathematics

Editors

J.H. Ewing
F.W. Gehring
P.R. Halmos

Springer Science+Business Media, LLC

Undergraduate Texts in Mathematics

- Anglin:** Mathematics: A Concise History and Philosophy.
Readings in Mathematics.
- Anglin/Lambek:** The Heritage of Thales.
Readings in Mathematics.
- Apostol:** Introduction to Analytic Number Theory. Second edition.
- Armstrong:** Basic Topology.
- Armstrong:** Groups and Symmetry.
- Axler:** Linear Algebra Done Right.
- Bak/Newman:** Complex Analysis. Second edition.
- Banchoff/Wermer:** Linear Algebra Through Geometry. Second edition.
- Berberian:** A First Course in Real Analysis.
- Brémaud:** An Introduction to Probabilistic Modeling.
- Bressoud:** Factorization and Primality Testing.
- Bressoud:** Second Year Calculus.
Readings in Mathematics.
- Brickman:** Mathematical Introduction to Linear Programming and Game Theory.
- Browder:** Mathematical Analysis: An Introduction.
- Buskes/van Rooij:** Topological Spaces: From Distance to Neighborhood.
- Cederberg:** A Course in Modern Geometries.
- Childs:** A Concrete Introduction to Higher Algebra. Second edition.
- Chung:** Elementary Probability Theory with Stochastic Processes. Third edition.
- Cox/Little/O'Shea:** Ideals, Varieties, and Algorithms. Second edition.
- Croom:** Basic Concepts of Algebraic Topology.
- Curtis:** Linear Algebra: An Introductory Approach. Fourth edition.
- Devlin:** The Joy of Sets: Fundamentals of Contemporary Set Theory. Second edition.
- Dixmier:** General Topology.
- Driver:** Why Math?
- Ebbinghaus/Flum/Thomas:** Mathematical Logic. Second edition.
- Edgar:** Measure, Topology, and Fractal Geometry.
- Elaydi:** Introduction to Difference Equations.
- Exner:** An Accompaniment to Higher Mathematics.
- Fine/Rosenberger:** The Fundamental Theory of Algebra.
- Fischer:** Intermediate Real Analysis.
- Flanigan/Kazdan:** Calculus Two: Linear and Nonlinear Functions. Second edition.
- Fleming:** Functions of Several Variables. Second edition.
- Foulds:** Combinatorial Optimization for Undergraduates.
- Foulds:** Optimization Techniques: An Introduction.
- Franklin:** Methods of Mathematical Economics.
- Gordon:** Discrete Probability.
- Hairer/Wanner:** Analysis by Its History.
Readings in Mathematics.
- Halmos:** Finite-Dimensional Vector Spaces. Second edition.
- Halmos:** Naive Set Theory.
- Hämmerlin/Hoffmann:** Numerical Mathematics.
Readings in Mathematics.
- Hijab:** Introduction to Calculus and Classical Analysis.
- Hilton/Holton/Pedersen:** Mathematical Reflections: In a Room with Many Mirrors.
- Iooss/Joseph:** Elementary Stability and Bifurcation Theory. Second edition.
- Isaac:** The Pleasures of Probability.
Readings in Mathematics.
- James:** Topological and Uniform Spaces.
- Jänich:** Linear Algebra.
- Jänich:** Topology.

(continued after index)

L. Christine Kinsey

Topology of Surfaces

With 276 Illustrations



Springer

L. Christine Kinsey
Department of Mathematics
Canisius College
Buffalo, NY 14208 USA

Editorial Board:

John H. Ewing
Department of Mathematics
Indiana University
Bloomington, IN 47405 USA

F.W. Gehring
Department of Mathematics
University of Michigan
Ann Arbor, MI 48109 USA

Paul R. Halmos
Department of Mathematics
Santa Clara University
Santa Clara, CA 95053 USA

Mathematics Subject Classifications (1991): 54-01, 55-01

Library of Congress Cataloging-in-Publication Data

Kinsey, L. Christine.

Topology of surfaces / L. Christine Kinsey.

p. cm. - (Undergraduate texts in mathematics)

Includes bibliographical references and index.

ISBN 978-1-4612-6939-7

ISBN 978-1-4612-0899-0 (eBook)

DOI 10.1007/978-1-4612-0899-0

(Berlin: acid-free)

I. Topology. I. Title. II. Series.

QA611.K47 1993

514-dc20

93-2605

Printed on acid-free paper.

© 1993 Springer Science+Business Media New York

Originally published by Springer-Verlag New York Berlin Heidelberg in 1993

Softcover reprint of the hardcover 1st edition 1993

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC, except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Henry Krell; manufacturing supervised by Vincent Scelta.

Photocomposed copy prepared from author's TeX files.

9 8 7 6 5 4 3 2 (Corrected second printing, 1997)

ISBN 978-1-4612-6939-7

SPIN 10540133

To Ray

Preface

“...that famous pedagogical method whereby one begins with the general and proceeds to the particular only after the student is too confused to understand even that anymore.”

Michael Spivak

This text was written as an antidote to topology courses such as Spivak describes. It is meant to provide the student with an experience in geometric topology. Traditionally, the only topology an undergraduate might see is point-set topology at a fairly abstract level. The next course the average student would take would be a graduate course in algebraic topology, and such courses are commonly very homological in nature, providing quick access to current research, but not developing any intuition or geometric sense. I have tried in this text to provide the undergraduate with a pragmatic introduction to the field, including a sampling from point-set, geometric, and algebraic topology, and trying not to include anything that the student cannot immediately experience. The exercises are to be considered as an integral part of the text and, ideally, should be addressed when they are met, rather than at the end of a block of material. Many of them are quite easy and are intended to give the student practice working with the definitions and digesting the current topic before proceeding. The appendix provides a brief survey of the group theory needed. The choice of topics addressed is my own, and fairly random at that, and was dictated by the fact that the text is used in a semester course. I often augment the text with articles from the general scientific and undergraduate math journals, which is a gentle way of getting the student to read and discuss mathematics.

This book originated as a set of lecture notes for a course in geometric topology at Vanderbilt University, and later at Canisius College. The present text has been used for a course taught to senior math majors. I have also taught a similar course for sophomores and juniors who have had a course in linear algebra but not in group theory, in which case I covered Chapters 1, 2, 4, 5, 6, 7, and 10.

This text was prepared using the \LaTeX macro package and the

macros supplied by Springer-Verlag New York. Portions of the text were prepared with the support of a Canisius College faculty fellowship. I would like to thank Metod Alif, Terry Bisson, Jim Catalano, Alex Douglas, Richard Escobales, Jim Huard, Mike Mihalik, Ray Mullican, Ian Putnam, John Ratcliffe, Tina Romance, Karl Schroeder, and Marlene Trzaska for many corrections, comments, and suggestions. Any mistakes remain my own.

L. Christine Kinsey
Canisius College

Contents

Preface	vii
Chapter 1. Introduction to topology	1
1.1. An overview	1
Chapter 2. Point-set topology in \mathbb{R}^n	7
2.1. Open and closed sets in \mathbb{R}^n	7
2.2. Relative neighborhoods	16
2.3. Continuity	19
2.4. Compact sets	26
2.5. Connected sets	29
2.6. Applications	31
Chapter 3. Point-set topology	37
3.1. Open sets and neighborhoods	37
3.2. Continuity, connectedness, and compactness	43
3.3. Separation axioms	47
3.4. Product spaces	48
3.5. Quotient spaces	52
Chapter 4. Surfaces	56
4.1. Examples of complexes	56
4.2. Cell complexes	60
4.3. Surfaces	67
4.4. Triangulations	75
4.5. Classification of surfaces	81
4.6. Surfaces with boundary	91
Chapter 5. The euler characteristic	94
5.1. Topological invariants	94
5.2. Graphs and trees	95
5.3. The euler characteristic and the sphere	101
5.4. The euler characteristic and surfaces	108
5.5. Map-coloring problems	113
5.6. Graphs revisited	119

Chapter 6. Homology	125
6.1. The algebra of chains	126
6.2. Simplicial complexes	135
6.3. Homology	139
6.4. More computations	151
6.5. Betti numbers and the euler characteristic	154
Chapter 7. Cellular functions	157
7.1. Cellular functions	157
7.2. Homology and cellular functions	165
7.3. Examples	170
7.4. Covering spaces	175
Chapter 8. Invariance of homology	183
8.1. Invariance of homology for surfaces	183
8.2. The Simplicial Approximation Theorem	187
Chapter 9. Homotopy	197
9.1. Homotopy and homology	197
9.2. The fundamental group	203
Chapter 10. Miscellany	212
10.1. Applications	212
10.2. The Jordan Curve Theorem	221
10.3. 3-manifolds	226
Chapter 11. Topology and calculus	235
11.1. Vector fields and differential equations in \mathbb{R}^n	235
11.2. Differentiable manifolds	248
11.3. Vector fields on manifolds	250
11.4. Integration on manifolds	257
Appendix: Groups	263
References	272
Index	274