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Volume 72

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Discrete Probability and Algorithms

With 7 Illustrations

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FOREWORD

This IMA Volume in Mathematics and its Applications

DISCRETE PROBABILITY AND ALGORITHMS

is based on the proceedings of two workshops, "Probability and Algorithms" and "The Finite Markov Chain Renaissance" that were an integral part of the 1993–94 IMA program on "Emerging Applications of Probability." We thank David Aldous, Persi Diaconis, Joel Spencer, and J. Michael Steele for organizing these workshops and for editing the proceedings. We also take this opportunity to thank the National Science Foundation, the Air Force Office of Scientific Research, the Army Research Office, and the National Security Agency, whose financial support made the workshop possible.

Avner Friedman

Willard Miller, Jr.

PREFACE

Discrete probability theory and the theory of algorithms have become close partners over the last ten years, though the roots of the partnership go back much longer. There are many reasons that underlie the coordination of these two fields, but some sense of the driving principles can be evoked by considerations like the following:

> When the use of a rule in an algorithm might lead to locking conflicts, randomization often provides a way to avoid stalemate.
>
> When a combinatorial object cannot be easily constructed, one can still often show the existence of the object by showing that under a suitable probability model such an object (or one close enough for appropriate modification) will exist with positive probability.
>
> When one needs to make a random uniform selection from an intractably large set, one can sometimes succeed by making clever use of a random walk (or other Markov chain) that has for its stationary measure the desired distribution.
>
> Finally, in many large systems that are driven by elements of chance, one often finds a certain steadiness that can be expressed by limit laws of probability theory and that can be exploited in the design of algorithms.

All of the chapters in this volume touch on one or more of these themes. The method of probabilistic construction is at the heart of the paper by Spencer and Tetali on Sidon sets as well as that of Godbole, Skipper, and Sunley, which traces its roots back to one of the first great successes of the "probabilistic method" — Erdős's pioneering analysis of the central Ramsey numbers.

The theme of steadiness in large random structures is evident in almost all of the volume's chapters, but it is made explicit in the paper by Fill and Dobrow on the move-to-front rule for self-organizing lists, the chapter by Yukich on Euclidean functionals (like the TSP), and in the paper by Steele that explores the limit theory that has evolved from the Erdős-Szekeres theorem on monotone subsequences. The chapter by Alon also shows how to find algorithmically useful "order in chaos" by developing a basic criterion of network connectivity in random graph models with unequal probabilities for edges.

The theme of "uniform selection by walking around" is perhaps most explicitly illustrated in the two chapters by Diaconis and Gangolli and Diaconis and Holmes. The first of these shows how one can use the ideas of the "Markov Chain Renaissance" to make progress on the difficult problem of the enumeration of integer tables with specified row sums and column sums. The second paper shows how new developments emerging from the Markov chain renaissance can be brought to bear on problems of concern

in statistics, computer science, and statistical mechanics. Aldous also makes a contribution in the thick of the new theory of finite Markov chains by showing in his chapter that one can simulate an observation from a chain's stationary distribution (quickly, though approximately)— all the while not knowing the transition probabilities of the chain except through the action of a "take a step from state x" oracle.

The two further chapters in this collection are surveys that call on all of the basic themes recalled above. They are also tightly tied to the central concerns of the theory of probabilistic complexity. The first of these is the survey of A. Karlin and P. Raghavan on random walks in undirected graphs—a notion that is present in many of the collection's chapters. The second is the survey by D. Welsh on randomized approximation schemes for Tutte-Gröthendieck invariants, which are remarkable polynomials whose values at special points give precise information about such basic graph theoretic problems as the number of connected subgraphs, the number of forest subgraphs, the number of acyclic orientations, and much more.

All the papers in this volume come from two Workshops, "Probability and Algorithms" and "The Finite Markov Chain Renaissance," that were held during the Special Year in Emerging Applications of Probability at the Institute for Mathematics and Its Applications at the University of Minnesota during the fall of 1993. The IMA provided a singularly congenial environment for productive scientific exchange, and with any luck the chapters of this volume will convey a sense of the excitement that could be felt in the progress that was reported in these IMA Workshops.

It is a pleasure to thank Avner Friedman, Willard Miller, Jr., and the IMA staff for their efficient organization of the workshops and the entire program, and to thank Patricia V. Brick for administering the preparation of this volume.

David Aldous
Persi Diaconis
Joel Spencer
J. Michael Steele

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