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Matrix Analysis

Springer

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Mathematics Subject Classification (1991): 15-01, 15A16, 15A45, 47A55, 65F15

Library of Congress Cataloging-in-Publication Data
Bhatia, Rajendra, 1952-
Matrix analysis / Rajendra Bhatia.
p. cm. – (Graduate texts in mathematics ; 169)
Includes bibliographical references and index.
ISBN 978-1-4612-6857-4 ISBN 978-1-4612-0653-8 (eBook)
DOI 10.1007/978-1-4612-0653-8
1. Matrices. I. Title. II. Series.
QA188.B485 1996
512.9'434–dc20 96-32217

Printed on acid-free paper.

Originally published by Springer-Verlag New York Berlin Heidelberg in 1997
Softcover reprint of the hardcover 1st edition 1997

Production managed by Victoria Evarretta; manufacturing supervised by Jeffrey Taub.
Photocomposed pages prepared from the author's LaTeX files.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-6857-4 SPIN 10524488

Preface

A good part of matrix theory is functional analytic in spirit. This statement can be turned around. There are many problems in operator theory, where most of the complexities and subtleties are present in the finite-dimensional case. My purpose in writing this book is to present a systematic treatment of methods that are useful in the study of such problems.

This book is intended for use as a text for upper division and graduate courses. Courses based on parts of the material have been given by me at the Indian Statistical Institute and at the University of Toronto (in collaboration with Chandler Davis). The book should also be useful as a reference for research workers in linear algebra, operator theory, mathematical physics and numerical analysis.

A possible subtitle of this book could be *Matrix Inequalities.* A reader who works through the book should expect to become proficient in the art of deriving such inequalities. Other authors have compared this art to that of cutting diamonds. One first has to acquire hard tools and then learn how to use them delicately.

The reader is expected to be very thoroughly familiar with basic linear algebra. The standard texts *Finite-Dimensional Vector Spaces* by P.R. Halmos and *Linear Algebra* by K. Hoffman and R. Kunze provide adequate preparation for this. In addition, a basic knowledge of functional analysis, complex analysis and differential geometry is necessary. The usual first courses in these subjects cover all that is used in this book.

The book is divided, conceptually, into three parts. The first five chapters contain topics that are basic to much of the subject. (Of these, Chapter 5 is more advanced and also more special.) Chapters 6 to 8 are devoted to perturbation of spectra, a topic of much importance in numerical analysis, physics and engineering. The last two chapters contain inequalities and perturbation bounds for other matrix functions. These too have been of broad interest in several areas.

In Chapter 1, I have given a very brief and rapid review of some basic topics. The aim is not to provide a crash course but to remind the reader of some important ideas and theorems and to set up the notations that are used in the rest of the book. The emphasis, the viewpoint, and some proofs may be different from what the reader has seen earlier. Special attention is given to multilinear algebra; and inequalities for matrices and matrix functions are introduced rather early. After the first chapter, the exposition proceeds at a much more leisurely pace. The contents of each chapter have been summarised in its first paragraph.

The book can be used for a variety of graduate courses. Chapters 1 to 4 should be included in any course on Matrix Analysis. After this, if perturbation theory of spectra is to be emphasized, the instructor can go on to Chapters 6,7 and 8. With a judicious choice of topics from these chapters, she can design a one-semester course. For example, Chapters 7 and 8 are independent of each other, as are the different sections in Chapter 8. Alternately, a one-semester course could include much of Chapters 1 to 5, Chapter 9, and the first part of Chapter 10. All topics could be covered comfortably in a two-semester course. The book can also be used to supplement courses on operator theory, operator algebras and numerical linear algebra. The book has several exercises scattered in the text and a section called Problems at the end of each chapter. An *exercise* is placed at a particular spot with the idea that the reader should do it at that stage of his reading and then proceed further. *Problems*, on the other hand, are designed to serve different purposes. Some of them are supplementary exercises, while others are about themes that are related to the main development in the text. Some are quite easy while others are hard enough to be contents of research papers. From Chapter 6 onwards, I have also used the problems for another purpose. There are results, or proofs, which are a bit too special to be placed in the main text. At the same time they are interesting enough to merit the attention of anyone working, or planning to work, in this area. I have stated such results as parts of the Problems section, often with hints about their solutions. This should enhance the value of the book as a reference, and provide topics for a seminar course as well. The reader should not be discouraged if he finds some of these problems difficult. At a few places I have drawn attention to some unsolved research problems. At some others, the existence of such problems can be inferred from the text. I hope the book will encourage some readers to solve these problems too.

While most of the notations used are the standard ones, some need a little explanation:

Almost all functional analysis books written by mathematicians adopt the convention that an inner product $\langle u, v \rangle$ is linear in the variable u and conjugate-linear in the variable v. Physicists and numerical analysts adopt the opposite convention, and different notations as well. There would be no special reason to prefer one over the other, except that certain calculations and manipulations become much simpler in the latter notation. If u and v are column vectors, then u^*v is the product of a row vector and a column vector, hence a number. This is the inner product of u and v. Combined with the usual rules of matrix multiplication, this facilitates computations. For this reason, I have chosen the second convention about inner products, with the belief that the initial discomfort this causes some readers will be offset by the eventual advantages. (Dirac's bra and ket notation, used by physicists, is different typographically but has the same idea behind it.)

The k-fold tensor power of an operator is represented in this book as $\otimes^k A$, the antisymmetric and the symmetric tensor powers as $\wedge^k A$ and $\vee^k A$, respectively. This helps in thinking of these objects as maps, $A \to \otimes^k A$, etc. We often study the variational behaviour of, and perturbation bounds for, functions of operators. In such contexts, this notation is natural.

Very often we have to compare two n-tuples of numbers after rearranging them. For this I have used a pictorial notation that makes it easy to remember the order that has been chosen. If $x = (x_1, \ldots, x_n)$ is a vector with real coordinates, then $x^\downarrow$ and $x^\uparrow$ are vectors whose coordinates are obtained by rearranging the numbers x_j in decreasing order and in increasing order, respectively. We write $x^\downarrow = (x_1^\downarrow, \ldots, x_n^\downarrow)$ and $x^\uparrow = (x_1^\uparrow, \ldots, x_n^\uparrow)$, where $x_1^\downarrow \geq \cdots \geq x_n^\downarrow$ and $x_1^\uparrow \leq \cdots \leq x_n^\uparrow$.

The symbol $|||\cdot|||$ stands for a unitarily invariant norm on matrices: one that satisfies the equality $|||UAV||| = |||A|||$ for all A and for all unitary U, V. A statement like $|||A||| \leq |||B|||$ means that, for the matrices A and B, this inequality is true simultaneously for all unitarily invariant norms. The supremum norm of A, as an operator on the space $\mathbb{C}^n$, is always written as $||A||$. Other norms carry special subscripts. For example, the Frobenius norm, or the Hilbert-Schmidt norm, is written as $||A||_2$. (This should be noted by numerical analysts who often use the symbol $||A||_2$ for what we call $||A||$.)

A few symbols have different meanings in different contexts. The reader's attention is drawn to three such symbols. If x is a complex number, $|x|$ denotes the absolute value of x. If x is an n-vector with coordinates $(x_1, \ldots, x_n)$, then $|x|$ is the vector $(|x_1|, \ldots, |x_n|)$. For a matrix A, the symbol $|A|$ stands for the positive semidefinite matrix $(A^*A)^{1/2}$. If J is a finite set, $|J|$ denotes the number of elements of J. A permutation on n indices is often denoted by the symbol σ. In this case, $\sigma(j)$ is the image of the index j under the map σ. For a matrix $A, \sigma(A)$ represents the spectrum of A. The trace of a matrix A is written as tr A. In analogy, if $x = (x_1, \ldots, x_n)$ is a vector, we write tr x for the sum Σx_j.

The words matrix and operator are used interchangeably in the book. When a statement about an operator is purely finite-dimensional in content, I use the word matrix. If a statement is true also in infinite-dimensional spaces, possibly with a small modification, I use either the word matrix or the word operator. Many of the theorems in this book have extensions to infinite-dimensional spaces.

Several colleagues have contributed to this book, directly and indirectly. I am thankful to all of them. T. Ando, J.S. Aujla, R.B. Bapat, A. Ben Israel, I. Ionascu, A.K. Lal, R.-C.Li, S.K. Narayan, D. Petz and P. Rosenthal read parts of the manuscript and brought several errors to my attention. Fumio Hiai read the whole book with his characteristic meticulous attention and helped me eliminate many mistakes and obscurities. Long-time friends and coworkers M.D. Choi, L. Elsner, J.A.R. Holbrook, R. Horn, F. Kittaneh, A. McIntosh, K. Mukherjea, K.R. Parthasarathy, P. Rosenthal and K.B. Sinha, have generously shared with me their ideas and insights. These ideas, collected over the years, have influenced my writing.

I owe a special debt to T. Ando. I first learnt some of the topics presented here from his Hokkaido University lecture notes. I have also learnt much from discussions and correspondence with him. I have taken a lot from his notes while writing this book.

The idea of writing this book came from Chandler Davis in 1986. Various logistic difficulties forced us to abandon our original plans of writing it together. The book is certainly the poorer for it. Chandler, however, has contributed so much to my mathematics, to my life, and to this project, that this is as much his book as it is mine.

I am thankful to the Indian Statistical Institute, whose facilities have made it possible to write this book. I am also thankful to the Department of Mathematics of the University of Toronto and to NSERC Canada, for several visits that helped this project take shape.

It is a pleasure to thank V.P. Sharma for his LaTeXtyping, done with competence and with good cheer, and the staff at Springer-Verlag for their help and support.

My most valuable resource while writing, has been the unstinting and ungrudging support from my son Gautam and wife Irpinder. Without that, this project might have been postponed indefinitely.

Rajendra Bhatia

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