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M.I. Freidlin A.D. Wentzell

Random Perturbations of Dynamical Systems

Second Edition

Translated by Joseph Szücs

With 33 Illustrations

Springer

M.I. Freidlin
Department of Mathematics
University of Maryland
College Park, MD 20742
USA

A.D. Wentzell
Department of Mathematics
Tulane University
New Orleans, LA 70118
USA

Joseph Szücs (*Translator*)
Texas A&M University at Galveston
P.O. Box 1675
Galveston, TX 77553
USA

Mathematics Subject Classification (1991): 58F30, 60F10, 60H25

Library of Congress Cataloging-in-Publication Data
Freĭdlin, M.I. (Mark Iosifovich)
Random perturbations of dynamical systems / Mark I. Freidlin, Alexander D. Wentzell ; [Joseph Szücs (translator)]. — 2nd ed.
p. cm. — (Grundlehren der mathematischen Wissenschaften ; 260)
"[Based on the] original Russian edition: Fluktuatsii v dinamicheskikh sistemakh pod deĭstviem malykh sluchaĭnykh vozmushcheniĭ, Nauka : Moscow, 1979"—T.p. verso.
Includes bibliographical references and index.
ISBN 978-1-4612-6839-0 ISBN 978-1-4612-0611-8 (eBook)
DOI 10.1007/978-1-4612-0611-8
1. Stochastic processes. 2. Perturbation (Mathematics)
I. Venttsel', A.D. II. Venttsel', A.D. Fluktuatsii v dinamicheskikh sistemakh pod deĭstviem malykh sluchaĭnykh vozmushcheniĭ. III. Title. IV. Series.
QA274.F73 1998
519.2—dc21 98-4686

Printed on acid-free paper.

Original Russian edition: *Fluktuatsii v Dinamicheskikh Sistemakh Pod Deistviem Malykh Sluchainykh Vozmushchenii*, Nauka: Moscow, 1979.

Production managed by Anthony Guardiola; manufacturing supervised by Joe Quatela.
Typeset by Asco Trade Typesetting Ltd., Hong Kong.

9 8 7 6 5 4 3 2 1

SPIN 10647749

Preface to the Second Edition

The first edition of this book was published in 1979 in Russian. Most of the material presented was related to large-deviation theory for stochastic processes. This theory was developed more or less at the same time by different authors in different countries. This book was the first monograph in which large-deviation theory for stochastic processes was presented. Since then a number of books specially dedicated to large-deviation theory have been published, including S. R. S. Varadhan [4], A. D. Wentzell [9], J.-D. Deuschel and D. W. Stroock [1], A. Dembo and O. Zeitouni [1]. Just a few changes were made for this edition in the part where large deviations are treated. The most essential is the addition of two new sections in the last chapter. Large deviations for infinite-dimensional systems are briefly considered in one new section, and the applications of large-deviation theory to wave front propagation for reaction-diffusion equations are considered in another one.

Large-deviation theory is not the only class of limit theorems arising in the context of random perturbations of dynamical systems. We therefore included in the second edition a number of new results related to the averaging principle. Random perturbations of classical dynamical systems under certain conditions lead to diffusion processes on graphs. Such problems are considered in the new Chapter 8. Some new results concerning fast oscillating perturbations of dynamical systems with conservation laws are included in Chapter 7. A few small additions and corrections were made in the other chapters as well. We would like to thank Ruth Pfeiffer and Fred Torcaso for their help in the preparation of the second edition of this book.

College Park, Maryland
New Orleans, Louisiana

M.I. Freidlin
A.D. Wentzell

Preface

Asymptotical problems have always played an important role in probability theory. In classical probability theory dealing mainly with sequences of independent variables, theorems of the type of laws of large numbers, theorems of the type of the central limit theorem, and theorems on large deviations constitute a major part of all investigations. In recent years, when random processes have become the main subject of study, asymptotic investigations have continued to play a major role. We can say that in the theory of random processes such investigations play an even greater role than in classical probability theory, because it is apparently impossible to obtain simple exact formulas in problems connected with large classes of random processes.

Asymptotical investigations in the theory of random processes include results of the types of both the laws of large numbers and the central limit theorem and, in the past decade, theorems on large deviations. Of course, all these problems have acquired new aspects and new interpretations in the theory of random processes.

One of the important schemes leading to the study of various limit theorems for random processes is dynamical systems subject to the effect of random perturbations. Several theoretical and applied problems lead to this scheme. It is often natural to assume that, in one sense or another, the random perturbations are small compared to the deterministic constituents of the motion. The problem of studying small random perturbations of dynamical systems has been posed in the paper by Pontrjagin, Andronov, and Vitt [1]. The results obtained in this article relate to one-dimensional and partly two-dimensional dynamical systems and perturbations leading to diffusion processes. Other types of random perturbations may also be considered; in particular, those arising in connection with the averaging principle. Here the smallness of the effect of perturbations is ensured by the fact that they oscillate quickly.

The contents of the book consists of various asymptotic problems arising as the parameter characterizing the smallness of random perturbations converges to zero. Of course, the authors could not consider all conceivable schemes of small random perturbations of dynamical systems. In particular, the book does not consider at all dynamical systems generated by random vector fields. Much attention is given to the study of the effect of perturbations on large time intervals. On such intervals small perturbations essentially influence the behavior of the system in general. In order to take account of this influence, we have to be able to estimate the probabilities of rare events, i.e., we need theorems on the asymptotics of probabilities of large deviations for random processes. The book studies these asymptotics and their applications to problems of the behavior of a random process on large time intervals, such as the problem of the limit behavior of the invariant measure, the problem of exit of a random process from a domain, and the problem of stability under random perturbations. Some of these problems have been formulated for a long time and others are comparatively new.

The problems being studied can be considered as problems of the asymptotic study of integrals in a function space, and the fundamental method used can be considered as an infinite-dimensional generalization of the well-known method of Laplace. These constructions are linked to contemporary research in asymptotic methods. In the cases where, as a result of the effect of perturbations, diffusion processes are obtained, we arrive at problems closely connected with elliptic and parabolic differential equations with a small parameter. Our investigations imply some new results concerning such equations. We are interested in these connections and as a rule include the corresponding formulations in terms of differential equations.

We would like to note that this book is being written when the theory of large deviations for random processes is just being created. There have been a series of achievements but there is still much to be done. Therefore, the book treats some topics that have not yet taken their final form (part of the material is presented in a survey form). At the same time, some new research is not reflected at all in the book. The authors attempted to minimize the deficiencies connected with this.

The book is written for mathematicians but can also be used by specialists of adjacent fields. The fact is that although the proofs use quite intricate mathematical constructions, the results admit a simple formulation as a rule.

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Sharpenings and Generalizations