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Alexandre M. Zagoskin

Quantum Theory of Many-Body Systems

Techniques and Applications

With 122 Illustrations

Springer

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To my parents

Preface

This book grew out of lectures that I gave in the framework of a graduate course in quantum theory of many-body systems at the Applied Physics Department of Chalmers University of Technology and Göteborg University (Göteborg, Sweden) in the years 1992–1995. Its purpose is to give a compact and self-contained account of basic ideas and techniques of the theory from the "condensed matter" point of view. The book is addressed to graduate students with knowledge of standard quantum mechanics and statistical physics. (Hopefully, physicists working in other fields may also find it useful.)

The approach is – quite traditionally – based on a quasiparticle description of many-body systems and its mathematical apparatus – the method of Green's functions. In particular, I tried to bring together all the main versions of diagram techniques for normal and superconducting systems, in and out of equilibrium (i.e., zero-temperature, Matsubara, Keldysh, and Nambu–Gor'kov formalisms) and present them in just enough detail to enable the reader to follow the original papers or more comprehensive monographs, or to apply the techniques to his own problems. Many examples are drawn from mesoscopic physics – a rapidly developing chapter of condensed matter theory and experiment, which deals with macroscopic systems small enough to preserve quantum coherence throughout their volume; this seems to me a natural ground to discuss quantum theory of many-body systems.

The plan of the book is as follows.

In Chapter 1, after a semi-qualitative discussion of the quasiparticle concept, Green's function is introduced in the case of one-body quantum theory, using Feynman path integrals. Then its relation to the $\mathcal{S}$-operator is established, and the general perturbation theory is developed based on operator formalism. Finally, the second quantization method is introduced.

Chapter 2 contains the usual zero-temperature formalism, beginning with the definition, properties and physical meaning of Green's function in the many-body system, and then builiding up the diagram technique of the perturbation theory.

In Chapter 3, I present equilibrium Green's functions at finite temperature, and then the Matsubara formalism. Their applications are discussed in relation to linear response theory. Then Keldysh technique is introduced as a means to handle essentially nonequilibrium situations, illustrated by an example of quantum conductivity of a point contact. This gives me an opportunity to discuss both Landauer and tunneling Hamiltonian approaches to transport in mesoscopic systems.

Finally, Chapter 4 is devoted to applications of the theory to the superconductors. Here the Nambu–Gor'kov technique is employed to describe superconducting phase transition, elementary excitations, and current-carrying state of a superconductor. Special attention is paid to the Andreev reflection and to transport in mesoscopic superconductor–normal metal–superconductor (SNS) Josephson junctions.

Each chapter is followed by a set of problems. Their solution will help the reader to obtain a better feeling for how the formalism works.

I did not intend to provide a complete bibliography, which would be far beyond the scope of this book. The *original papers* are cited when the results they contain are either recent or not widely known in the context, and in a few cases where interesting results would require too lengthy a derivation to be presented in full detail (those sections are marked by a star*). For references on more traditional material I have referred the reader to existing **monographs** or **reviews**.

For a course in quantum many-body theory based on this book I would suggest the following tentative schedule:[1]

Lecture 1 (Sect. 1.1); Lecture 2 (Sect. 1.2.1); Lecture 3 (Sect. 1.2.2, 1.2.3); Lecture 4 (Sect. 1.3); Lecture 5 (Sect. 1.4); Lecture 6 (Sect. 2.1.1); Lecture 7 (Sect. 2.1.2); Lecture 8 (Sect. 2.1.3, 2.1.4); Lecture 9 (Sect. 2.2.1, 2.2.2); Lecture 10 (Sect. 2.2.3); Lectures 11–12 (Sect. 2.2.4); Lecture 13 (Sect. 3.1); Lecture 14 (Sect. 3.2); Lecture 15 (Sect. 3.3); Lecture 16 (Sect. 3.4); Lecture 17 (Sect. 3.5); Lecture 18 (Sect. 3.6); Lecture 19 (Sect. 3.7); Lecture 20 (Sect. 4.1); Lecture 21 (Sect. 4.2); Lecture 22 (Sect. 4.3.1, 4.3.2); Lecture 23 (Sect. 4.3.3, 4.3.4); Lecture 24 (Sect. 4.4.1, 4.4.2); Lectures 25–26 (Sect. 4.4.3–5); Lecture 27 (Sect. 4.5.1); Lecture 28 (Sect. 4.5.2–4); Lecture 29 (Sect. 4.6).

Acknowledgments

I am deeply grateful to Professor R. Shekhter, collaboration with whom in preparing and giving the course on quantum theory of many-body system significantly influenced this book.

I wish to express my sincere thanks to the Institute for Low Temperature Physics and Engineering (Kharkov, Ukraine) and Professor I.O. Kulik, who first taught me

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[1] Based on a "two hours" (90 min) lecture length.

what condensed matter theory is about; to the Applied Physics Department of Chalmers University of Technology and Göteborg University (Göteborg, Sweden) and Professor M. Jonson, and to the Physics and Astronomy Department of the University of British Columbia (Vancouver, Canada) and Professor I. Affleck for support and encouragement.

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Last, but not least, I am grateful to my wife Irina for her unwavering support and for actually starting this project by the comment, "Well, if you are spending this much time on preparing these handouts, you should rather be writing a book," and to my daughter Ekaterina for critical appreciation of the illustrations.

Vancouver, British Columbia *Alexandre M. Zagoskin*

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