



Progress in Theoretical Computer Science

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Subrecursive

Programming

Systems

Complexity & Succinctness

Springer Science+Business Media, LLC

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Library of Congress Cataloging-in-Publication Data

Royer, James S.

Subrecursive programming systems : complexity & succinctness /
James S. Royer, John Case.

p. cm. - (Progress in theoretical computer science)

Includes bibliographical references and index.

ISBN 978-1-4612-6680-8 ISBN 978-1-4612-0249-3 (eBook)

DOI 10.1007/978-1-4612-0249-3

1. Electronic digital computers--Programming. I. Case, John,
1942- . II. Title. III. Series.

QA76.R7 1994

94-26443

005.13'1--dc20

CIP

Printed on acid-free paper

© Springer Science+Business Media New York 1994
Originally published by Birkhäuser Boston in 1994
Softcover reprint of the hardcover 1st edition 1994



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ISBN 978-1-4612-6680-8

Typeset by the authors.

9 8 7 6 5 4 3 2 1

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When I struggle to be terse, I end by being obscure.

Horace, *Ars Poetica*