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Dynamic Programming for Impulse Feedback and Fast Controls

The Linear Systems Case

Springer

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Preface

The text of this book deals with an important class of modern control problems motivated by applied issues, namely, those that are governed by controls of impulsive nature. Its first part describes dynamic processes under controls that include delta functions. The emphasis is on formalizing the theory of impulse open-loop target control, but not only. The further text deals with impulsive feedback—the theory of closed-loop controlled trajectories that may have jumps of coordinates caused by delta functions on their way to the end-point target. Described are the optimized dynamics of such systems under "ordinary impulse" controls and also in the presence of unknown but bounded disturbances. Further on considered are controlled dynamics under state constraints and closed-loop processes under incomplete on-line information on the system coordinates. The latter issue involves the theory of closed-loop observability that deals with on-line state estimation under disturbances of impulsive type. Indicated are duality properties between state-constrained control and state estimation under impulsive inputs. Emphasized are two types of duality—in the sense of mathematical optimization, between primal and dual variables, and in the sense of system theory—between solutions to problems of state control and state estimation. The second part of the book deals with controls described not only by ordinary delta functions but also by generalized functions of higher orders that include delta functions and their higher derivatives. The mathematics of dynamic systems under such controls includes specific behavior of linear controlled systems. Namely, if the input controls are linear combinations of delta functions and their derivatives, then they may solve the two-point boundary problems in zero time (+0). Related mathematical techniques are thoroughly explained in the text for both open-loop and closed-loop versions. Special attention is devoted to state-constrained systems under high-order impulsive control inputs and to state estimation of systems subjected to unknown but bounded high-order impulsive disturbances. Formulated also is a generalized duality principle between high-order impulse control and state estimation under high-order impulses. But impulses of any order are ideal constructions. Their application requires additional schemes that would include some types of computable procedures of regularization. This yields the last chapters of the book that describe physically realizable solutions and the theory of realizable the so-called fast controls. Such controls are designed by solving a problem with double constraints: the constraint that yields impulses combined with a hard bound on the values of the control. Fast controls are “ordinary functions” that may be selected as such that allow to solve two-point boundary problems in arbitrary small “nano”-time. The physical realization of impulse controls is achieved using fast functions. A bibliography of related prior published literature is given in the sequel followed by an Appendix on nonlinear systems with impulsive inputs. The mathematical level of the book presumes knowledge of advanced calculus and functional analysis, linear algebra and differential equations, elements of set-valued calculus, and basic computational methods. The authors express their gratitude to F. Allgower, K. Astrom, T. Basar, P. Kokotovic, A. Krener, A. A. Kurzhanskiy, A. Lindquiust, G. Leitmann, I. Mitchell, A. Rantzer, and P. Varaiya for their valuable discussions on the topics of this book thus helping to finalize its contents.

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Contents

Notations

$\mathbb{R}^n$	n-dimensional real vector space
$\mathbb{R}^{m\times n}$	linear space of $m \times n$-matrices
A^T	Transpose of matrix A
e^A	Matrix exponential
$BV([a,b];\mathbb{R}^n)$	Space of functions of bounded variation with values in $\mathbb{R}^n$
$\text{Var}_{[a,b]}f(\cdot)$	Variation of function f over interval $[a,b]$
$\mathbf{1}_{\mathscr{A}}(x)$	Indicator function of the set $\mathscr{A}$, equal to 1 when $x \in \mathscr{A}$ and 0 when $x \notin \mathscr{A}$
$\mathscr{I}(x\vert\mathscr{A})$	Indicator function of the set $\mathscr{A}$, equal to 0 when $x \in \mathscr{A}$ and ∞ when $x \notin \mathscr{A}$
$\chi(t)$	Heaviside's function, equal to 0 when $x \leq 0$ and 1 when $x > 0$
$\delta(t)$	Delta function, generalized derivative of Heaviside's function
$\mathscr{B}_{\Vert\cdot\Vert}$	Unit ball in norm$\Vert\cdot\Vert$
$f'(x\mid\xi)$	Directional derivative of $f(x)$ along direction ξ
$d(x,\mathscr{A})$	Distance from point x to set $\mathscr{A}$
$\langle x,y\rangle$	Scalar product of vectors x and y
$\langle f,x\rangle$	Linear functional f applied to vector x
$L_p[a,b]$	Space of functions with Lebesgue-integrable pth power
$\Vert x\Vert$	Norm of vector x
$\dot{x}(t)$	Derivative of $x(t)$ with respect to time t
$f^*(p)$	Fenchel conjugate of function $f(x)$
conv $\mathscr{A}$	Convex hull of set $\mathscr{A}$
conv $f(\cdot)$	Convex hull of function $f(\cdot)$ (equal to $f^{**}(x)$)
$\rho(p\mid\mathscr{A})$	Support function of set $\mathscr{A}$ in direction p
$L^\perp$	Orthogonal complement to space L
f_x	Derivative of function $f(x)$ with respect to variable x
dom f	Domain of function f (set of x such that $f(x) < \infty$)
$\partial^- f(x)$	Subgradient of function f at point x
$\partial_C f(x)$	Clarke subgradient of function f at point x